Introduction to Stellarators – Part I

PPPL Graduate Summer School 2021

(August 16th, 2021)

Adelle Wright

Email: awright@pppl.gov

Reference material

An Introduction to Stellarators

From magnetic fields to symmetries and optimization

Lise-Marie Imbert-Gérard, Elizabeth J. Paul, Adelle M. Wright

- L.-M. Imbert-Gerard, E. J. Paul and A. M. Wright (2019+).
- https://arxiv.org/abs/1908.05360
- A self-contained introduction covering the basic theoretical building blocks for modelling 3D magnetic fields, with applications to fusion device optimisation and design.
- No physics background assumed.
- Coming soon(-ish) in book form.

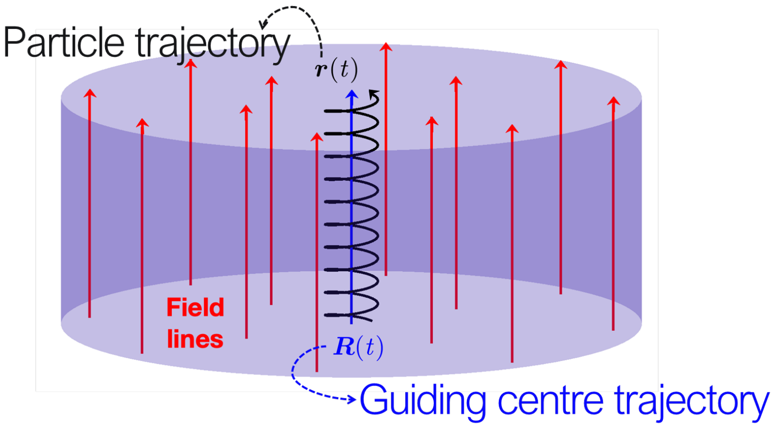
Contents

- Symmetry and magnetic confinement
- A selected history of stellarators (and PPPL)
- Techniques for stellarator design
- Towards a stellarator pilot plant

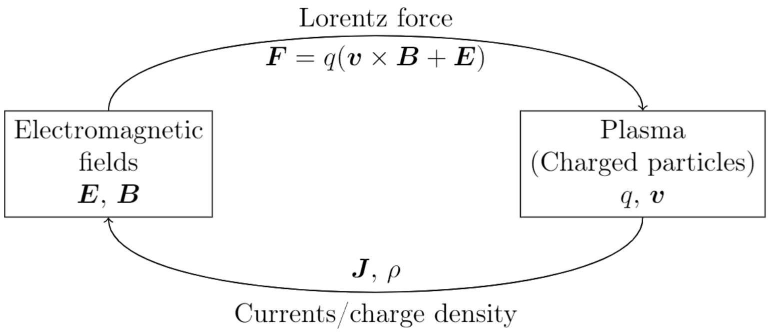
Symmetry and magnetic confinement

Charged particles in magnetic fields

- The gyration of charged particles about magnetic field lines is the basis of magnetic confinement fusion.

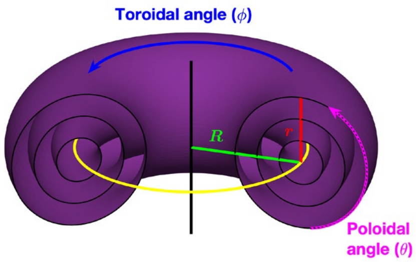


- Electromagnetic fields are described by Maxwell's equations while individual particle motion follows Newton's law ($\boldsymbol{F} = m\boldsymbol{a}$).

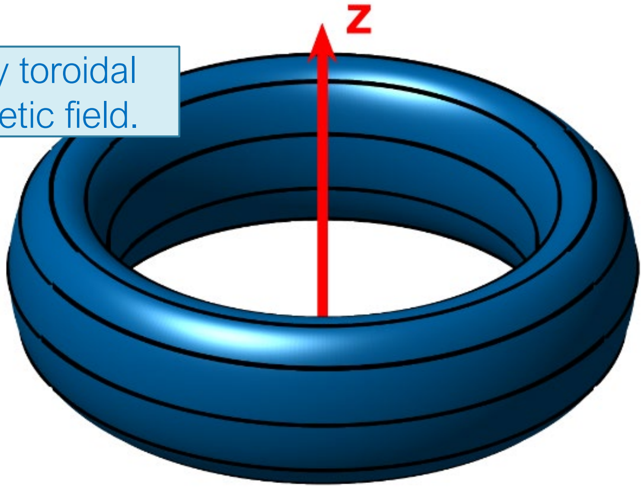


Particle confinement and non-zero rotational transform

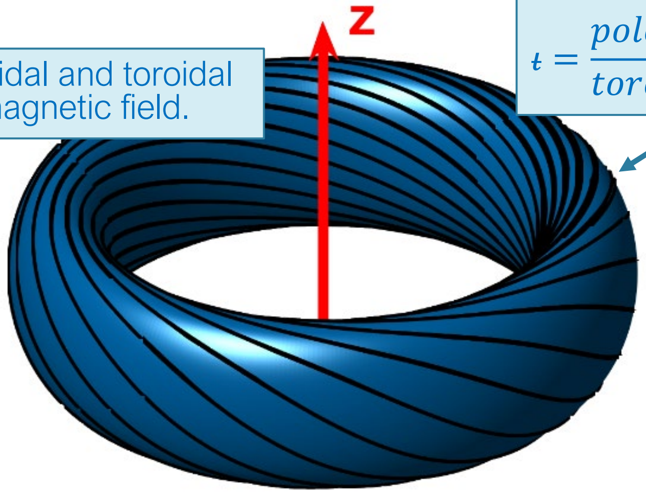
- In toroidal geometry, $\nabla\mathbf{B}$ causes particles to drift across magnetic field lines and eventually out of the plasma.
- A poloidal magnetic field (B_θ) is required for confinement.
- $B_\theta \neq \mathbf{0}$ causes magnetic field lines to twist → non-zero rotational transform (ι =poloidal/toroidal transits).



Purely toroidal magnetic field.



Poloidal and toroidal magnetic field.

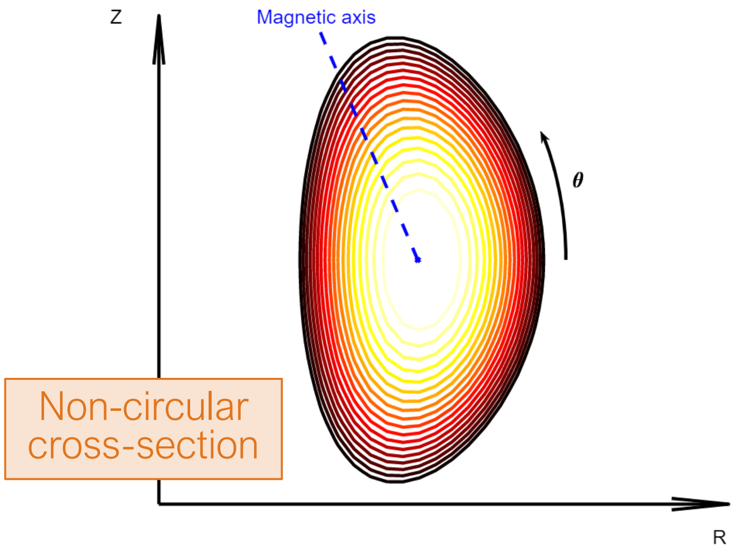


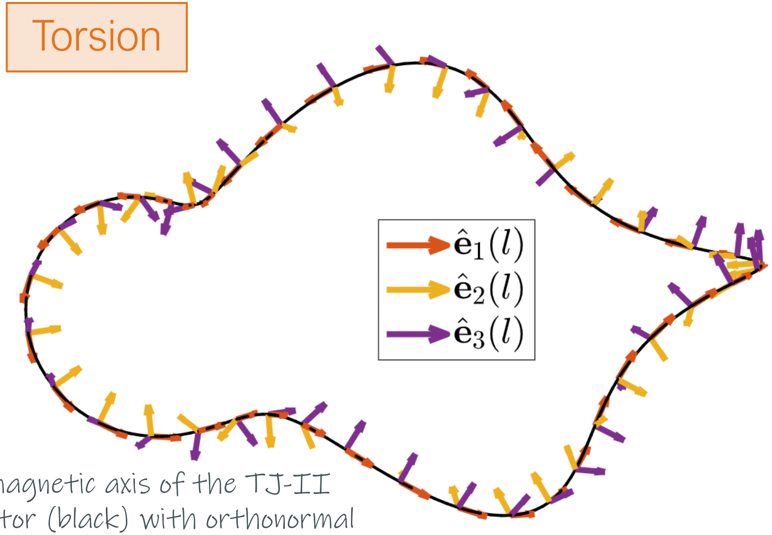
$$\iota = \frac{poloidal\ transits}{toroidal\ transits}$$

Generating rotational transform

There are two ways to generate rotational transform:

1. It can be induced by driving a toroidal current in the plasma.
2. Geometric effects: Change in ellipticity (non-circular toroidal cross-section) and torsion in the magnetic axis.





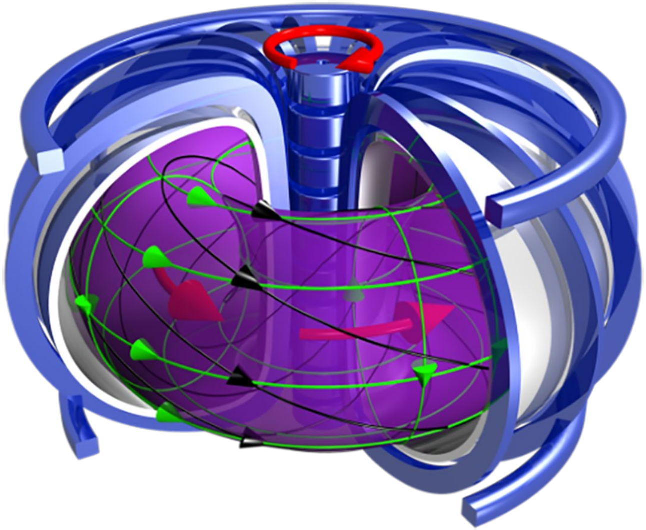
The magnetic axis of the TJ-II stellarator (black) with orthonormal Frenet-Serret unit vectors.

Generating rotational transform via geometric effects requires variations in the toroidal direction, ϕ:

- Under the assumption of axisymmetry ($\partial\phi \rightarrow 0$), rotational transform can only be produced by driving toroidal current.

Tokamaks *have* to drive large toroidal currents:

- This produces magnetic fields that are good at confining pressure.
- But leads to disruptive macroscopic instabilities.



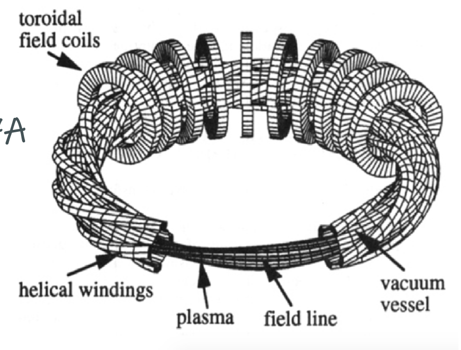
Generating rotational transform in stellarators

Stellarators generate rotational transform using geometric effects:

- This leads to intrinsically “3D” (i.e., nonaxisymmetric) devices.
- Avoids having to drive large toroidal currents.
- The confining magnetic field must be produced by external coils.
- Several approaches to stellarator coils have been tried.

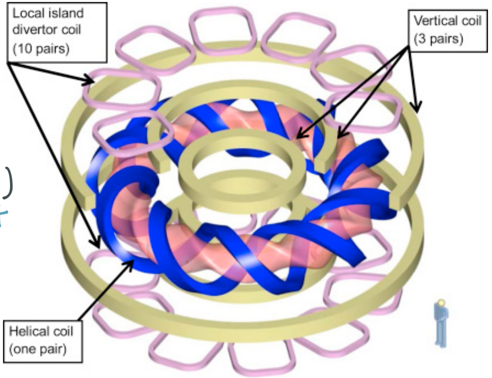
Classes of stellarator coils

Classical:



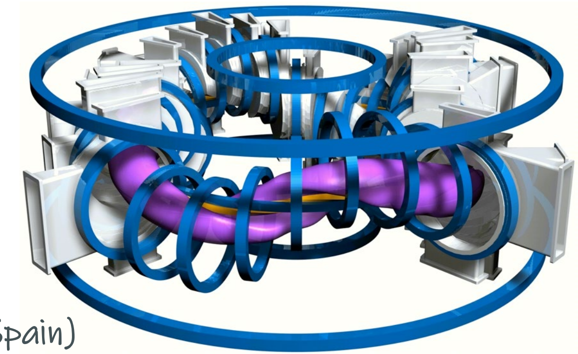
Wendelstein 7A (Germany)
1975-1985

Heliotron:



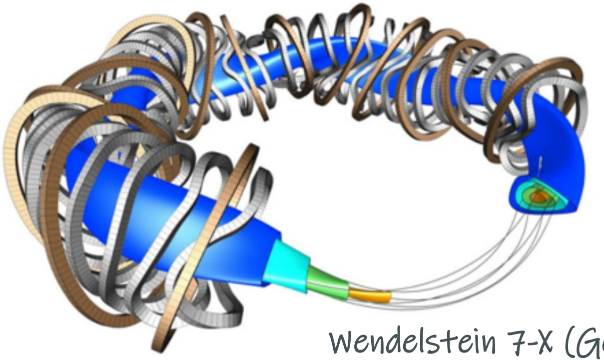
Large Helical Device (Japan)
1998-present

Heliac:



TJ-II (Spain)
1998-present

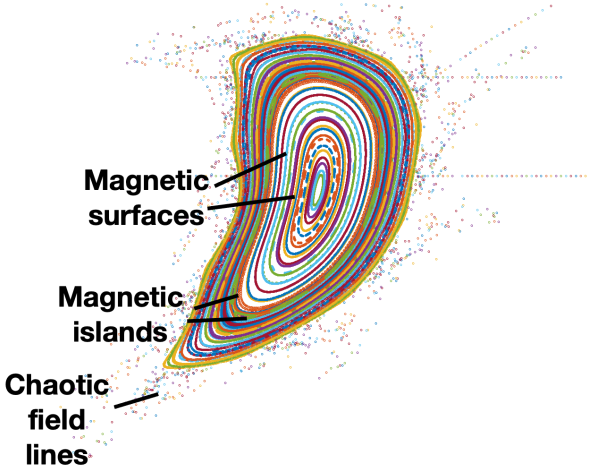
Modular coils:

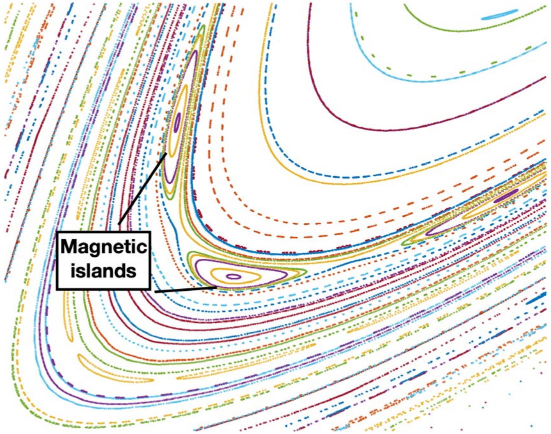


Wendelstein 7-X (Germany)
2015-present

Symmetry and magnetic field structure

Depending on underlying symmetries, magnetic fields can admit different topological structures:





- Maintaining a large number of magnetic surfaces is desirable for confinement, whereas islands and chaos are often to be avoided.
- More on magnetic coordinates on Tuesday [LMIG Day 2 2:30 pm].

Continuous symmetries and fusion plasma performance

- Noether's theorem → continuous symmetries are good.
- By generating rotational transform with externally driven currents, tokamaks preserve axisymmetry.
- This guarantees the existence of continuously nested flux surfaces, which is good for confinement.
- Coils for tokamaks are comparatively simple to design and manufacture. Less complexity reduces cost and lowers project risk.
- However, currents drive instabilities. Tokamak plasmas are very dynamic, posing challenges for macroscopic control and long timescale operation.

Symmetry breaking and fusion plasma performance

- Stellarators use geometric effects to generate rotational transform, which necessarily breaks axisymmetry ($\partial\phi \nrightarrow 0$).

- Stellarators do not have a continuous symmetry → continuously nested flux surfaces no longer guaranteed [LMIG, Day 4 Thursday 11:30am].

- However, some discrete symmetries still preserved:
  - Field periodicity (N-fold rotation symmetry).

- This also motivates the search for other approximate or local symmetries:
  - Quasisymmetry [EJP, Day 5 1 pm].

Hidden symmetries and fusion energy (2018-present)

- The Simons Collaboration on Hidden Symmetries and Fusion Energy is a multi-disciplinary, multi-institutional project led by Princeton University and funded by the Simons Foundation.
- The Collaboration brings together diverse expertise from **over 10 institutions across 3 continents**, spanning plasma physics, optimisation and dynamical systems theory, partial differential equations and high-performance computing.
- The project simultaneously aims to address the fundamental mathematical challenges associated with stellarator physics, while impacting stellarator design.
- The project has delivered SIMSOPT, a new open-source software framework for stellarator optimisation.

Symmetry-breaking and geometric complexity

+1

Because stellarators don't need to drive toroidal current, they are generally less susceptible to macroscopic, current-driven instabilities. This is good for steady-state operation.

-1

Since strong shaping is required for confinement and must be generated by external coils, stellarators are geometrically complex → challenging to design.

Increased sensitivity to coil configurations → tight engineering and construction tolerances increases project cost and risk. Sometimes prohibitively.

Symmetry-breaking and coil complexity

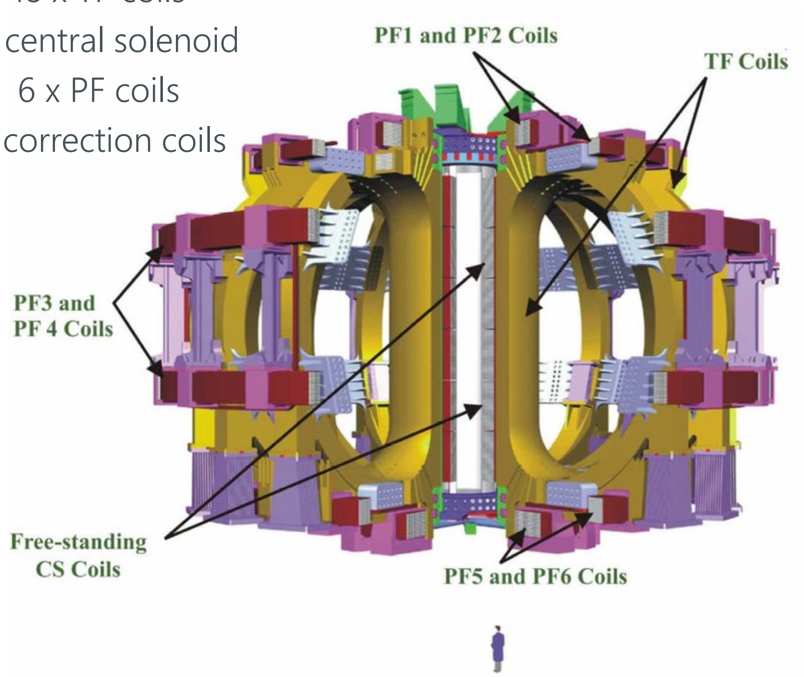
ITER coils:

18 x TF coils

1 x central solenoid

6 x PF coils

+ correction coils



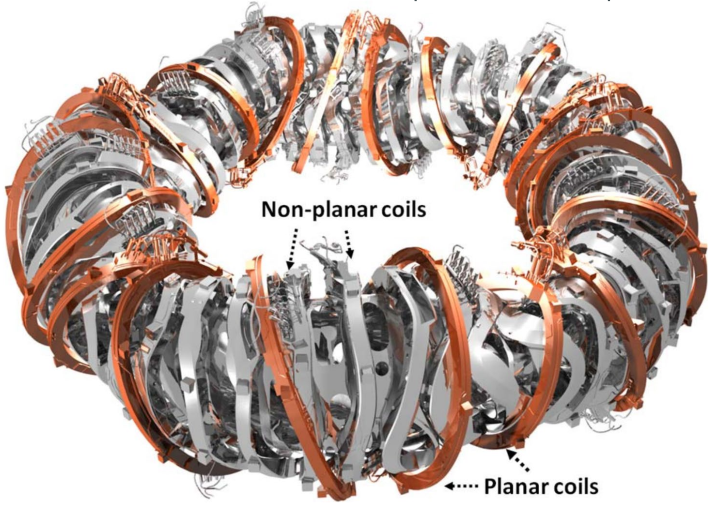
(From V. Tanna, FZKA (2006))

Wendelstein 7-X coils:

50 x nonplanar

20 x planar coils

Grouped in five equal modules



(From K. Risse et al., IEEE Transactions on Applied Superconductivity 26.4 (2016))

Example: Coil set for one module in W7-X

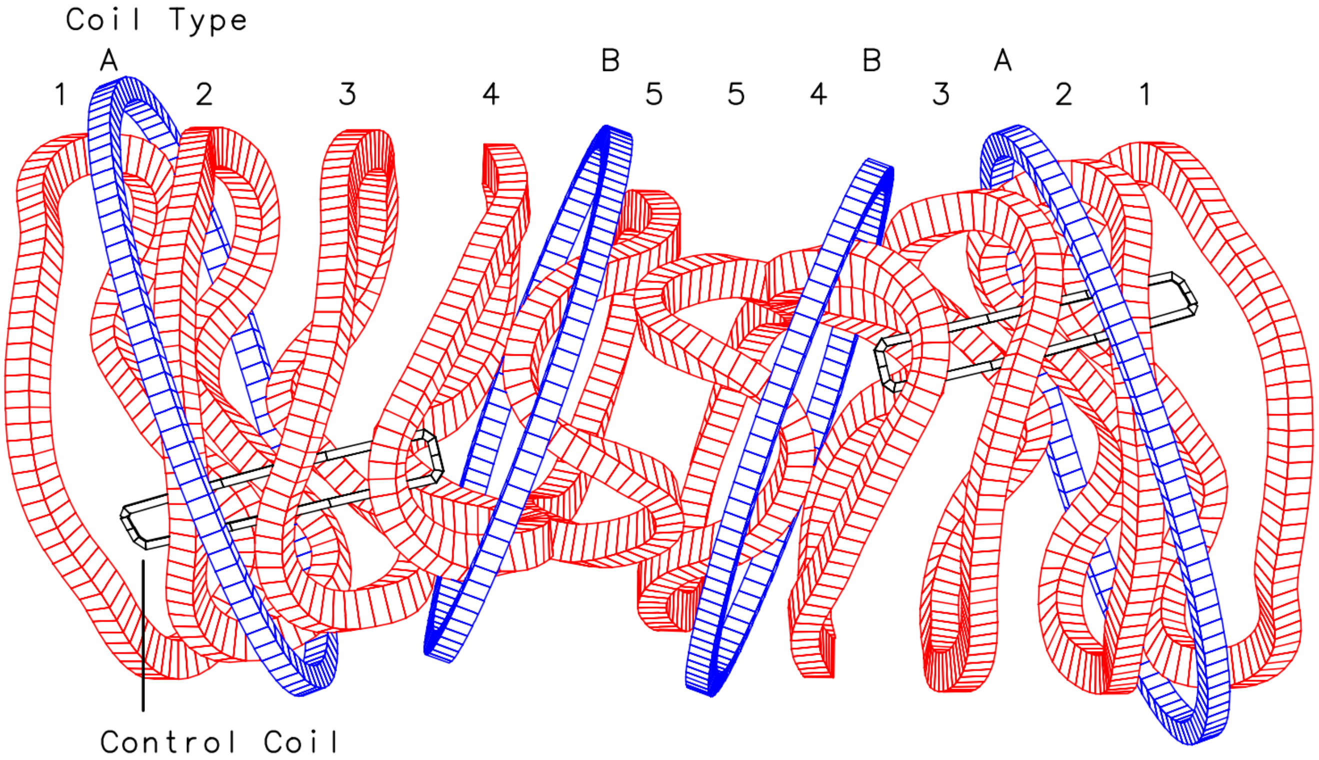


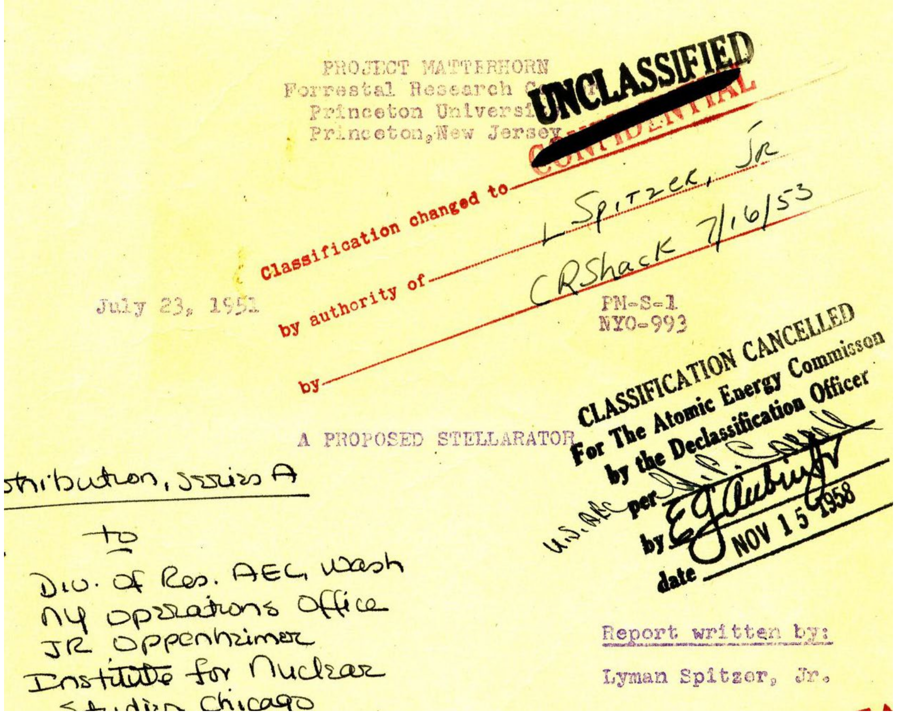
Figure 1. One field period of the W7-X coil system with modular (1 to 5) and planar (A and B) coil types.

(From J. Geiger et al, Plasma Phys. Control. Fusion 57 014004 (2015))

A selected history of stellarators (and PPPL)

The first stellarator concept (1951)

- Lyman Spitzer Jr. (1914-1997) had many good ideas, including the stellarator concept, which he proposed in 1951:


PROJECT MATTERHORN
Forrestal Research C
Princeton Universi
Princeton, New Jersey

UNCLASSIFIED
CONFIDENTIAL

Classification changed to L Spitzer, Jr
by authority of CRShack 7/16/53
by

July 23, 1951

PM-S-1
NYO-993

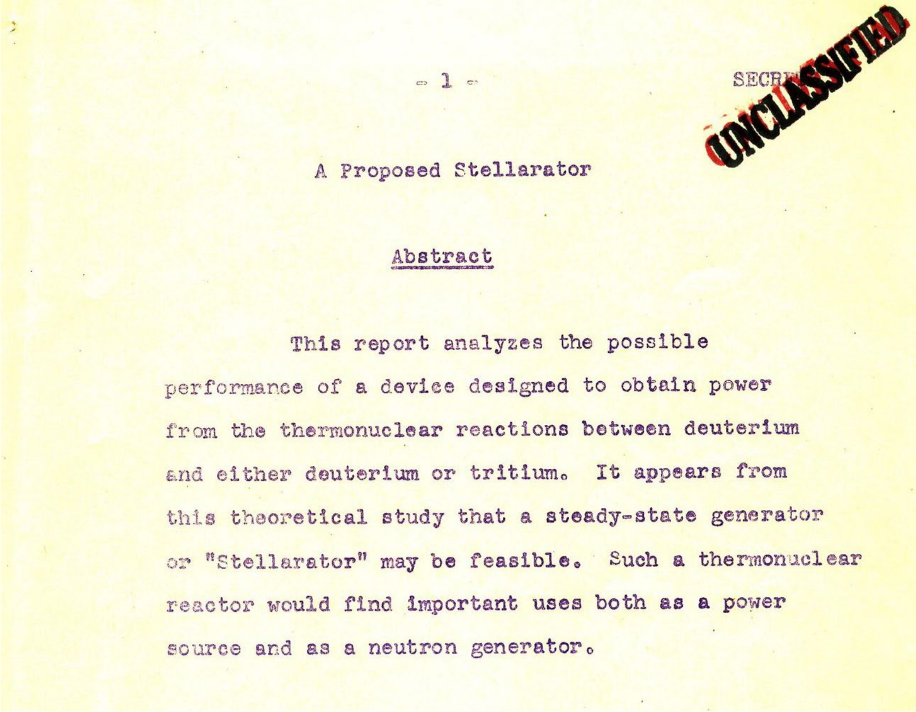
A PROPOSED STELLARATOR

CLASSIFICATION CANCELLED
For The Atomic Energy Commission
by the Declassification Officer
per
by
date NOV 15 1958

stribution, issued A
to
Div. of Res. AEC, Wash
NY operations Office
JR Oppenheimer
Institute for Nuclear
Studies Chicago

Report written by:

Lyman Spitzer, Jr.


- 1 -

SECR

UNCLASSIFIED

A Proposed Stellarator

Abstract

This report analyzes the possible performance of a device designed to obtain power from the thermonuclear reactions between deuterium and either deuterium or tritium. It appears from this theoretical study that a steady-state generator or "Stellarator" may be feasible. Such a thermonuclear reactor would find important uses both as a power source and as a neutron generator.

Project Matterhorn

- Spitzer's ideas led to the creation of Project Matterhorn (1951-1958), which was the code name for controlled thermonuclear research at Princeton University.
- Project Matterhorn was created, supported by The US Atomic Energy Commission and Princeton University.
- Early research in Project Matterhorn focused on stellarators.

Series I: PM-S Reports Content List

SELECT ITEMS	TITLE	DATE
	A Proposed Stellarator	1951 July 23
	Survey of Possible Plasma Oscillations in the Stellarator	1951 July 31
	Particle Orbits in a Low-Density Stellarator	1951 October 1
	Magnetic Fields and Particle Orbits in a High-Density Stellarator	1952 January 28
	Some Properties of Rotational Transforms	1952 February 18
	On the Ionization and Heating of a Plasma	1953 March 27
	Design and Construction of a Model A Stellarator	1953 March 17
	The Controlled Release of Thermonuclear Energy	1953 April 14
	Preliminary Experimental Results with the Model A Stellarator	1953 May 27
	On the Pulse Method of Ionization and Heating of a Plasma	1953 October 7
	Design of Correction Coils for the Model B Stellarator	1954 January
	Large-Scale Plasma Instability in the Stellarator	1954 April
	Heating of a Plasma by Magnetic Pumping	1954 May
	Problems of the Stellarator as a Useful Power Source	1954 August 1
	Orientation Lectures	1954 September 24
	The Steady State Plasma Equations for the Stellarator Under Diffusion	1955 May 25
	Magnetic Field Design for Stellarator Scallops	1956 May 26
	A Conceptual Design of the Model C Stellarator	1956 February 1
	Confining Ionized Plasma with Helical Magnetic Fields	1956 May 25

Early insights

- Spitzer and Project Matterhorn anticipated several of the key problems that continue to pose a challenge:
  - Limitations of ohmic heating and thus the need for auxiliary heating schemes.
  - Divertors and the need to protect the plasma from impurities caused by sputtering from the wall.
  - Vessel and coil forces.
- The work on Project Matterhorn also revealed several key insights that are essential to today's fusion devices including:
  - The need for rotational transform to provide confinement.
  - Role of magnetic shear for MHD stability.

The first stellarator concepts (early 1950s)

- In the first stellarators, torsion of the magnetic axis was used to generate rotational transform.
- This led to the "figure 8" and "racetrack" designs.

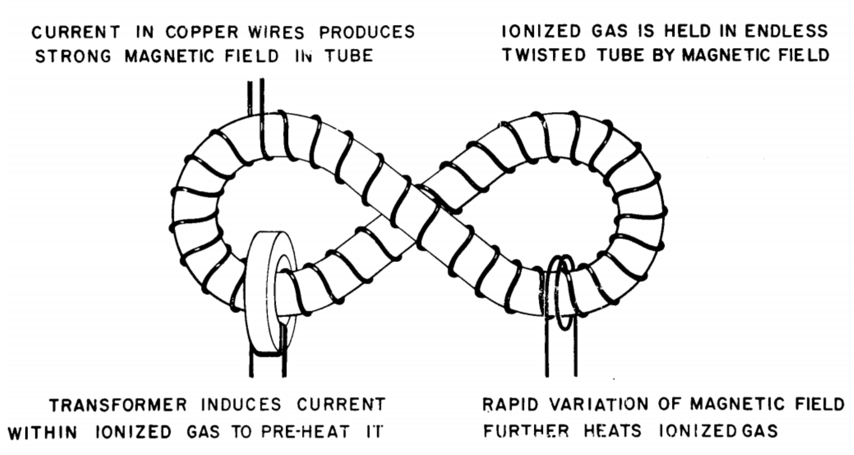
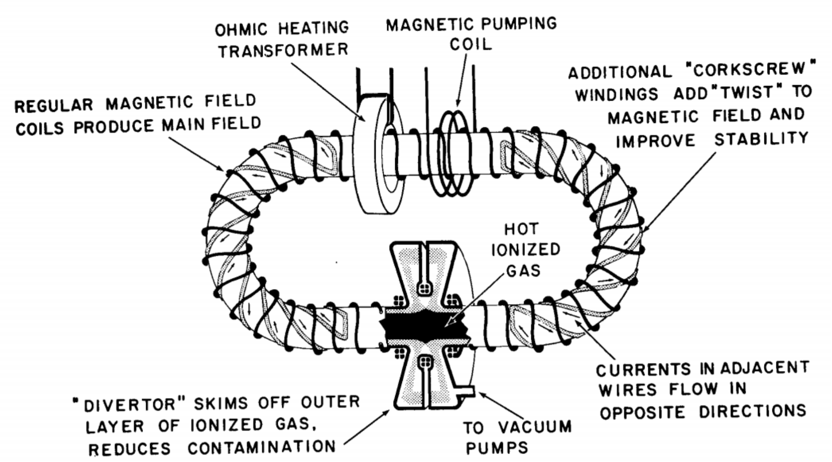


FIG. 3: SCHEMATIC FIGURE-8 STELLARATOR

FIG. 4: SCHEMATIC "RACETRACK" STELLARATOR


Stellarators at Princeton in the classified years (1951-1958)

- Several stellarator experiments were built during Project Matterhorn:

Model A (operational in 1953) (L=3.5 m, B~0.1 T)	Low-field, table-top device to demonstrate confinement of electrons.
Model B (operational in 1954) (a=0.05 m, L=4.5 m)	Initial issues with impurities and coil forces. Rebuilt and facilitated development of diagnostic techniques (spectroscopy and microwave methods). Verified Kruskal limit and observed tearing modes.
Model B-2 (early-mid 1950s) (a=0.05 m, L=6 m)	Built to study magnetic pumping. The device suffered from interchange instabilities and precipitated research on MHD stability. Led to the realization that magnetic shear is important for stabilisation → development of helical windings.
Model B-64 (operational in 1955) (B~1.8 T)	Developed to investigate modular approach to device construction. Originally called $B8^2$. If successful, would have been a prolific neutron source. To keep it a secret, AEC security office renamed the device, B-64. Demonstrated efficacy of divertors for impurity control.
Model B-3 (operational in 1958) (a=0.05 m, L=4.68 m, B~4 T, τ~10 μs)	Designed to study impurity control and successfully measured wall recycling.
Model B-65	Rebuilt from B-64 and verified rotational transform could be generated using helical windings.

(See T. H. Stix, Journal of Plasma Fusion Research 1.3 (1998))

Model A and B stellarators (1950s)

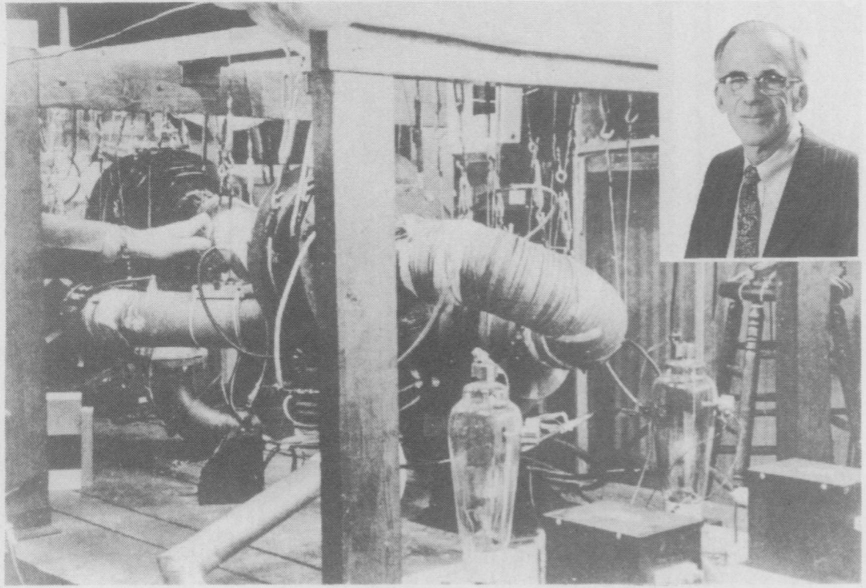


Fig. 1. Lyman Spitzer, Jr. (insert) and the Model A Stellarator.

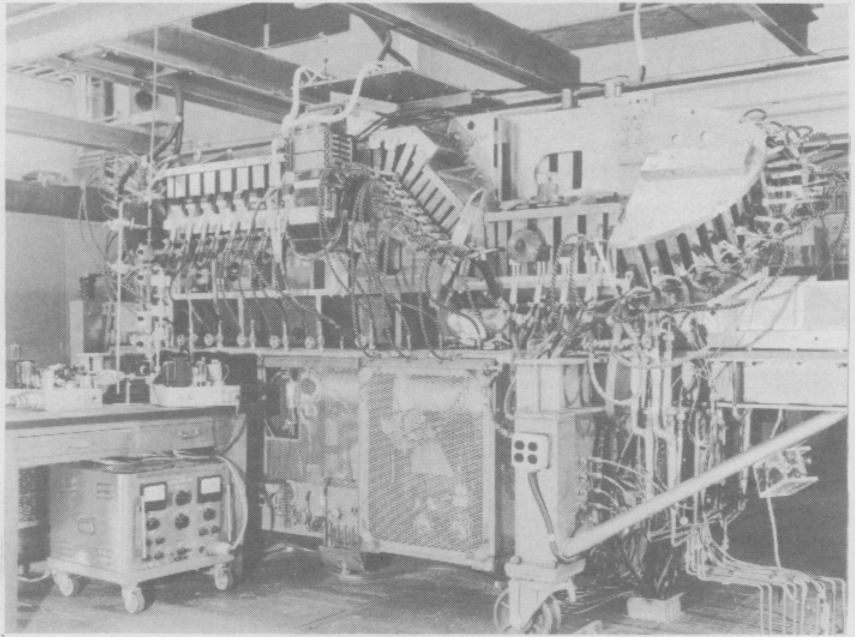
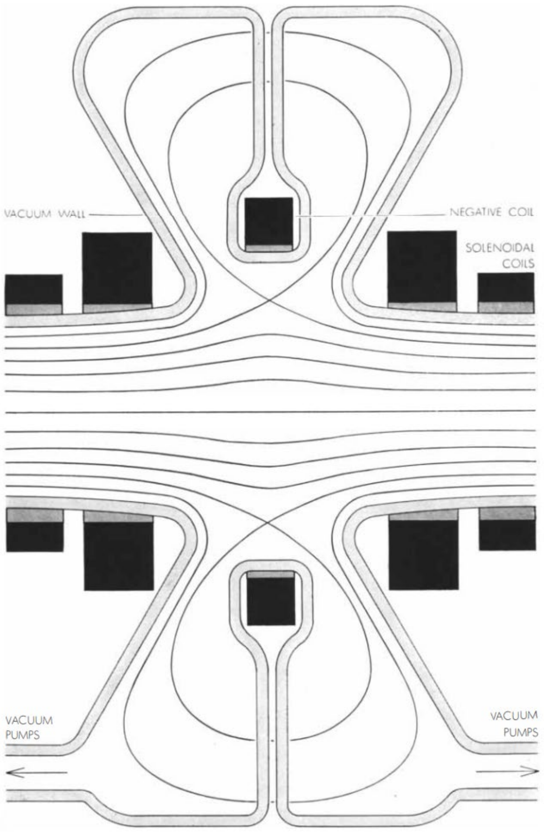


Fig. 2. The Model B-3 Stellarator.

(From J. L. Johnson et al. IEEE Trans. Plasma Science PS-9.4 (1981))

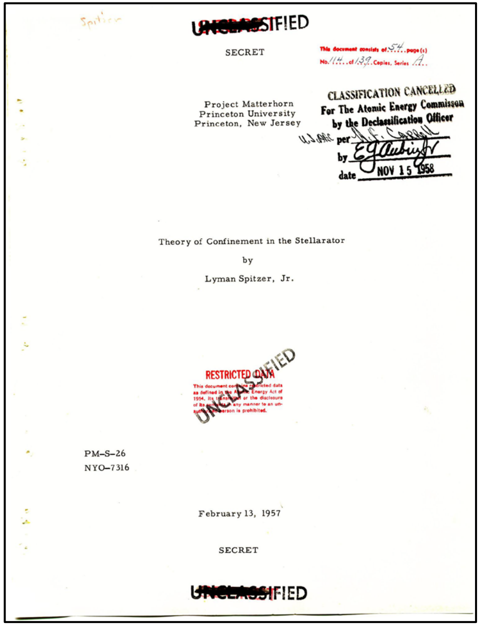
Early stellarator divertor concept (1950s)



DIVERTOR helps prevent gas particles from striking walls of tube. Magnetic lines near the walls are bent out into side chamber surrounding tube at one point. Particles traveling along these lines are swept into the chamber and pumped out of the system. This diagram is a cross section of the divertor; the black blocks are coils that bend the lines.

(LHS: From L. Spitzer, Scientific American, 199.4 (1958), pp. 28–35.)

- Work on the Model A and B stellarators led to the realisation that a larger device was needed to reach relevant temperatures.
- This informed the on-going design work towards the Model C stellarator and a fusion power plant study.


UNCLASSIFIED

SECRET

This document consists of 54 page(s) No. 114 of 139 Copies, Series A.

Project Matterhorn
Princeton University
Princeton, New Jersey

CLASSIFICATION CANCELLED
For The Atomic Energy Commission
by the Declassification Officer
per
by
date NOV 15 1958

Theory of Confinement in the Stellarator

by

Lyman Spitzer, Jr.

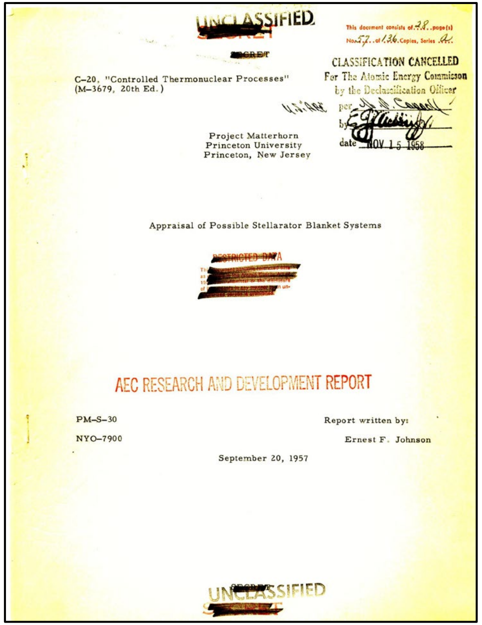
RESTRICTED DATA
UNCLASSIFIED

PM-S-26
NYO-7316

February 13, 1957

SECRET

UNCLASSIFIED


UNCLASSIFIED [redacted]

This document consists of 38 page(s) No. 57 of 136 Copies, Series A.

[redacted]

CLASSIFICATION CANCELLED
For The Atomic Energy Commission
by the Declassification Officer
per
by
date NOV 15 1958

C-20, "Controlled Thermonuclear Processes"
(M-3679, 20th Ed.)

Project Matterhorn
Princeton University
Princeton, New Jersey

Appraisal of Possible Stellarator Blanket Systems

[redacted]

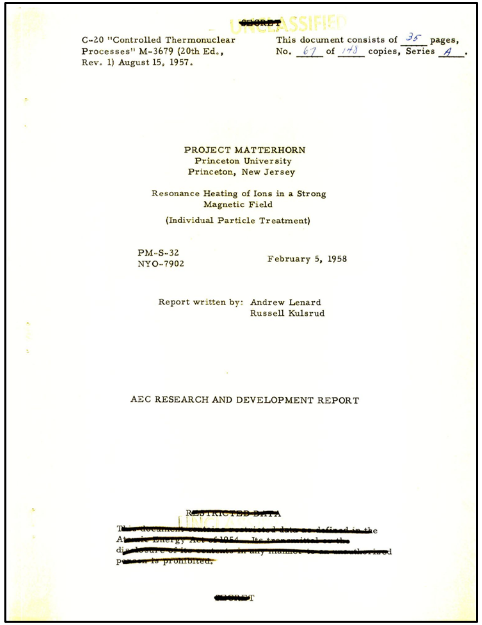
AEC RESEARCH AND DEVELOPMENT REPORT

PM-S-30
NYO-7900

Report written by:
Ernest F. Johnson

September 20, 1957

UNCLASSIFIED [redacted]


UNCLASSIFIED

C-20 "Controlled Thermonuclear Processes" M-3679 (20th Ed., Rev. 1) August 15, 1957.

This document consists of 35 pages, No. 67 of 143 copies, Series A.

PROJECT MATTERHORN
Princeton University
Princeton, New Jersey

Resonance Heating of Ions in a Strong Magnetic Field
(Individual Particle Treatment)

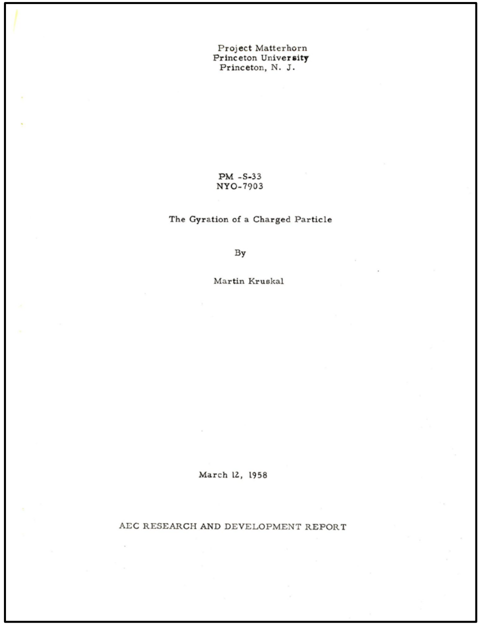
PM-S-32
NYO-7902

February 5, 1958

Report written by: Andrew Lenard
Russell Kulsrud

AEC RESEARCH AND DEVELOPMENT REPORT

[redacted]


Project Matterhorn
Princeton University
Princeton, N. J.

PM-S-33
NYO-7903

The Gyration of a Charged Particle

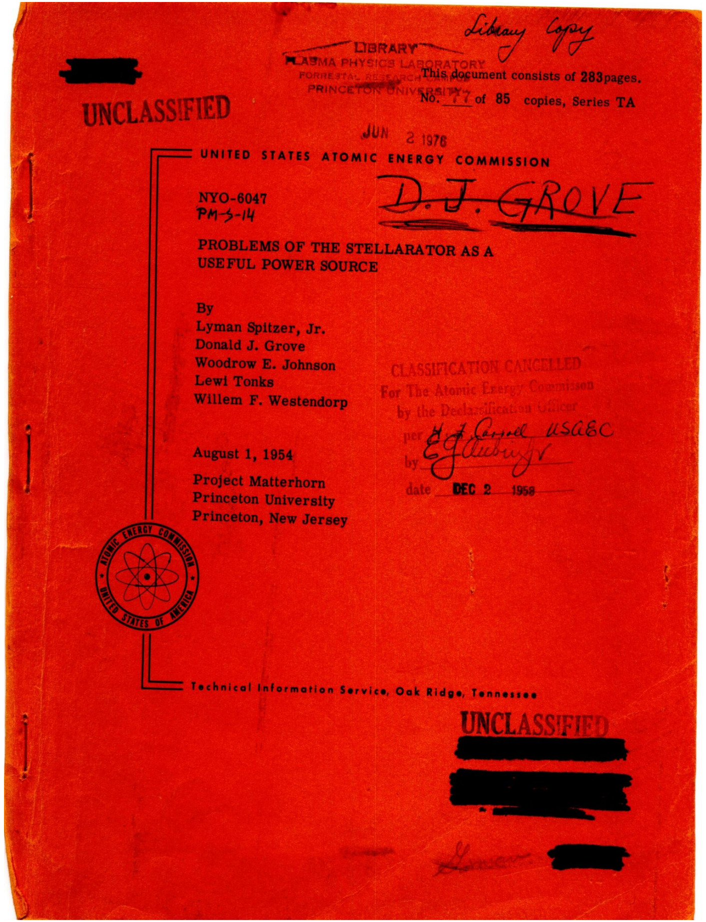
By

Martin Kruskal

March 12, 1958

AEC RESEARCH AND DEVELOPMENT REPORT

Model D – A stellarator power plant concept (circa 1954)



[redacted]

Library Copy

This document consists of 283 pages.
No. 77 of 85 copies, Series TA

UNCLASSIFIED

UNITED STATES ATOMIC ENERGY COMMISSION

NYO-6047
PM-S-14

D.J. GROVE

PROBLEMS OF THE STELLARATOR AS A USEFUL POWER SOURCE

By
Lyman Spitzer, Jr.
Donald J. Grove
Woodrow E. Johnson
Lewi Tonks
Willem F. Westendorp

CLASSIFICATION CANCELLED
For The Atomic Energy Commission
by the Declassification Officer
per H.J. Carroll USAEC
by
date DEC 2 1958

August 1, 1954

Project Matterhorn
Princeton University
Princeton, New Jersey

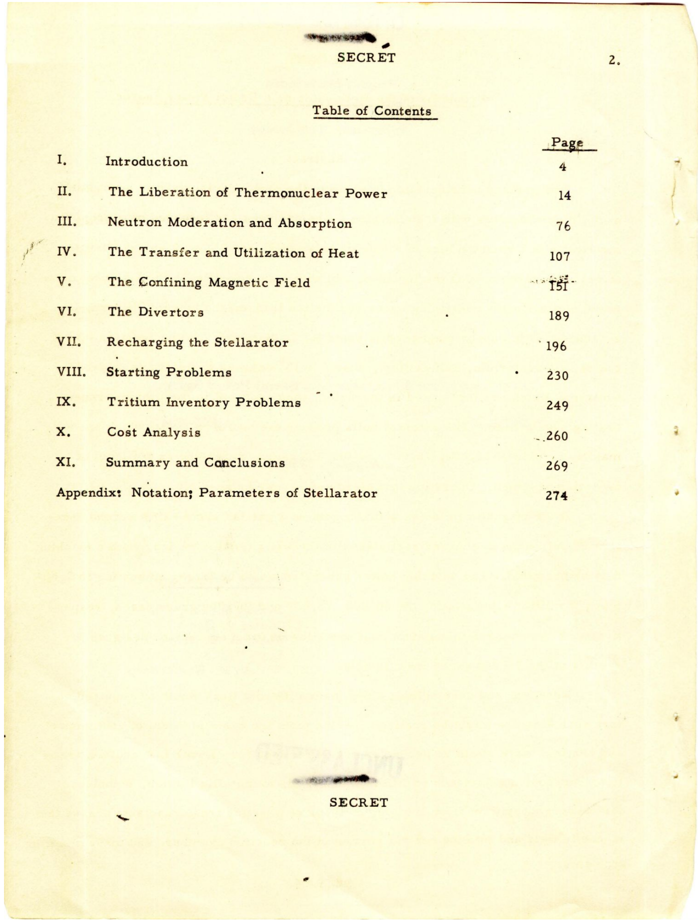
ATOMIC ENERGY COMMISSION
UNITED STATES OF AMERICA

Technical Information Service, Oak Ridge, Tennessee

UNCLASSIFIED

[redacted]

[redacted]



[redacted]

SECRET 2.

Table of Contents

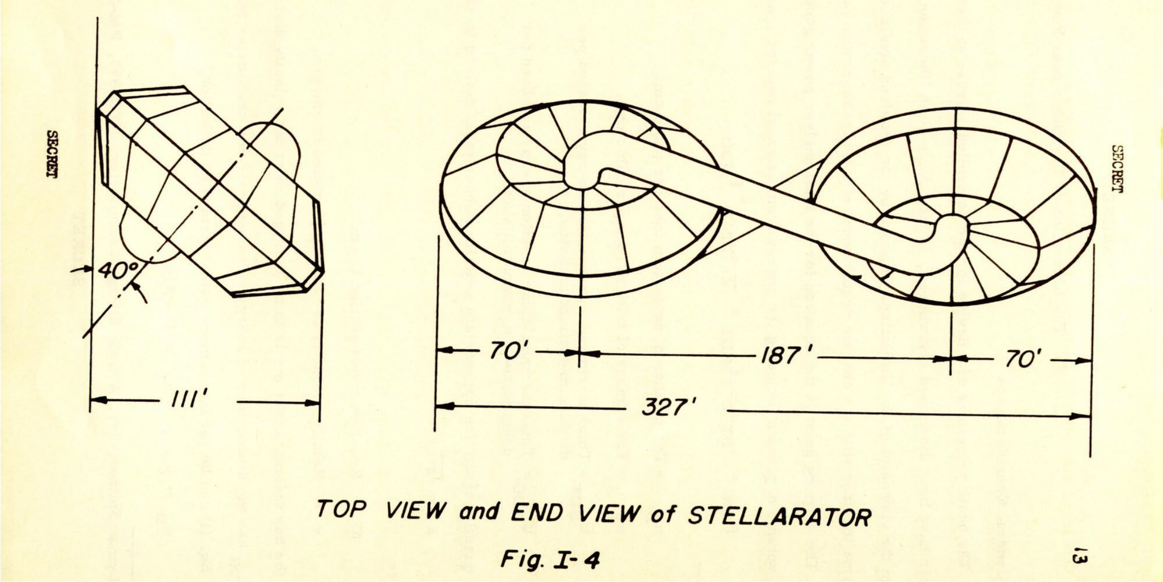
		Page
I.	Introduction	4
II.	The Liberation of Thermonuclear Power	14
III.	Neutron Moderation and Absorption	76
IV.	The Transfer and Utilization of Heat	107
V.	The Confining Magnetic Field	151
VI.	The Divertors	189
VII.	Recharging the Stellarator	196
VIII.	Starting Problems	230
IX.	Tritium Inventory Problems	249
X.	Cost Analysis	260
XI.	Summary and Conclusions	269
Appendix:	Notation; Parameters of Stellarator	274

[redacted]

SECRET

Even in 1954, key considerations for a fusion power plant did not differ substantially from the questions being examined today.

Model D – A stellarator power plant concept (1954)



TOP VIEW and END VIEW of STELLARATOR

Fig. I-4

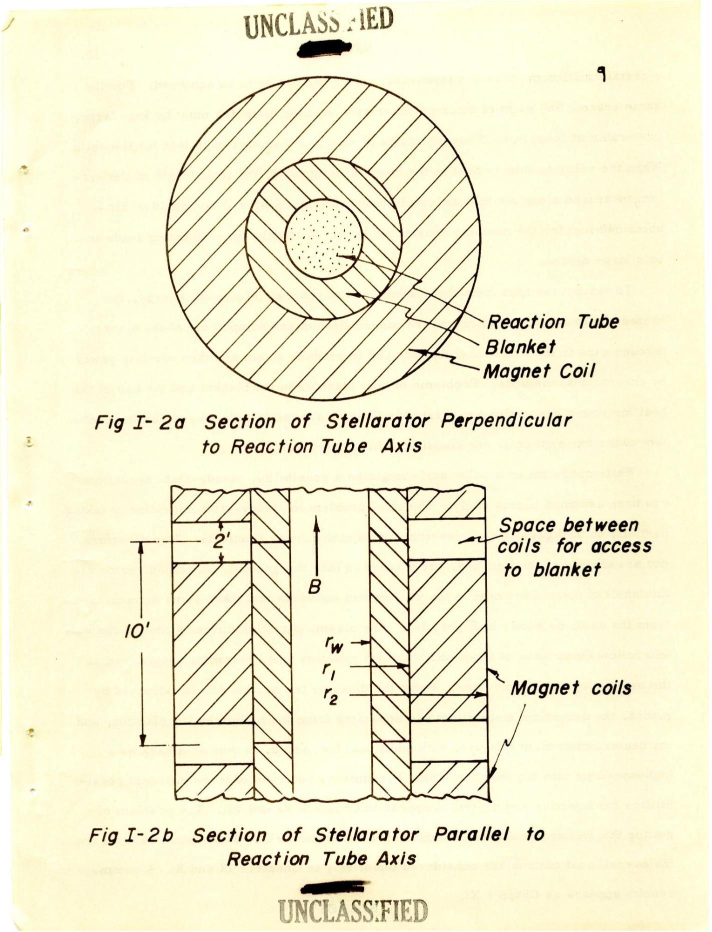


Fig I-2a Section of Stellarator Perpendicular to Reaction Tube Axis

Fig I-2b Section of Stellarator Parallel to Reaction Tube Axis

Model D – Design parameters (1954)

Appendix B. – Stellarator Design Parameters

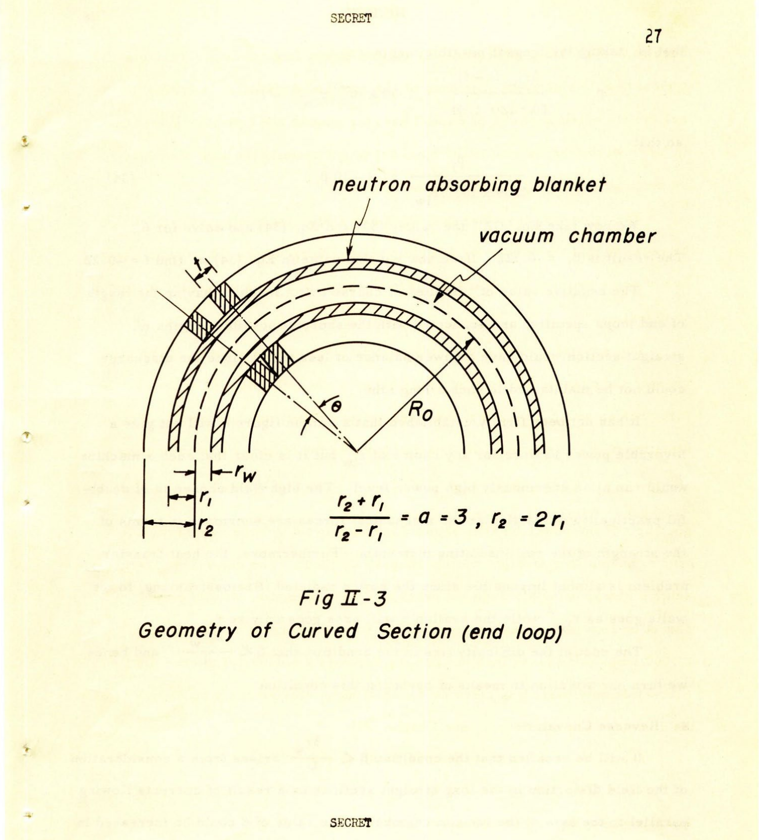
I. Quantities common to all three cases of maximum magnetic field and all sections

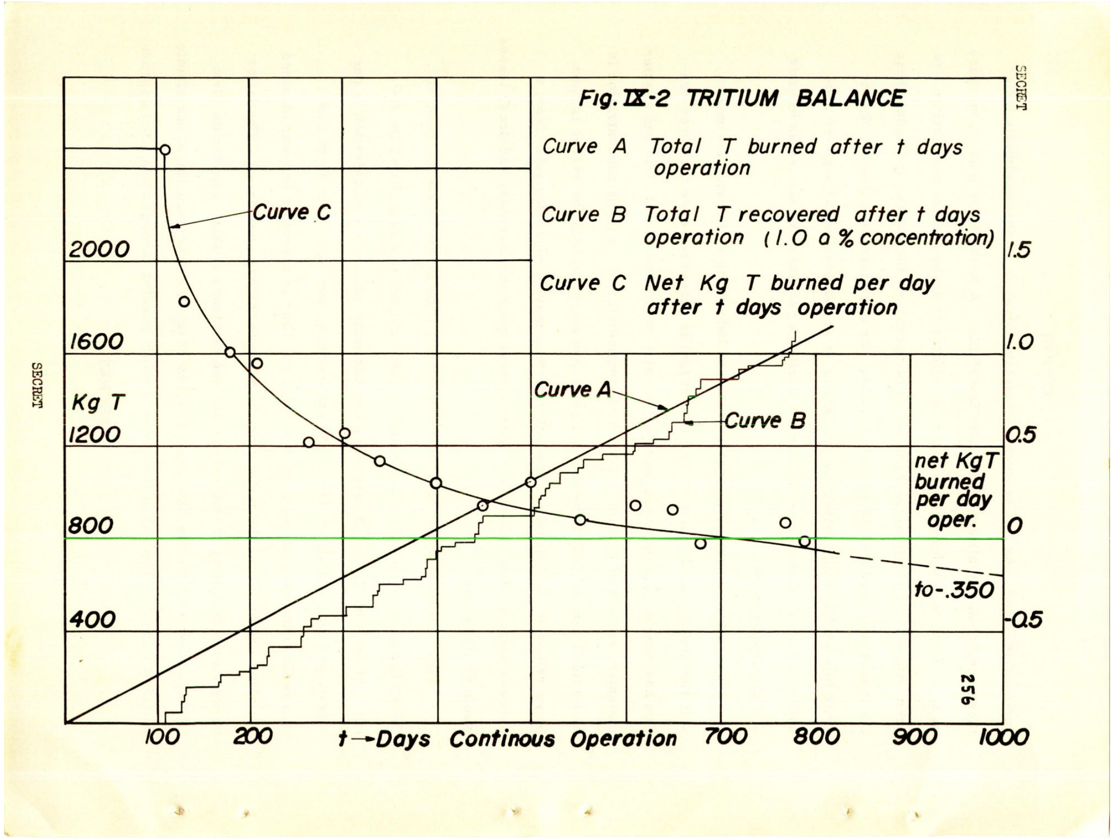
Plasma temperature	1×10^{8} degrees K.
Reaction coefficients	α_{DT} 8.0×10^{-17} $cm^3 sec^{-1}$
	α_{DD} 7.0×10^{-19} $cm^3 sec^{-1}$
Gas composition	50 % D, 50 % T μ = 0.5
Reaction tube surface area	6.22×10^{6} cm^2
Reaction tube total volume	1.92×10^{8} cm^3
Total axial length	540 feet
Overall stellarator dimensions	11 x 11 x 327 feet
Divertor surface area (for 2)	1.38×10^{8} cm^2
Divertor volume (for 2)	2.4×10^{10} cm^3
Neutron absorbing blanket	2 feet = 61 cm.
Total weight of copper for coils	3.92×10^{7} lbs.
Maximum copper stresses (100 kg case)	26,500 psi.
Maximum insulation stresses (100 kg case)	7800 psi.
Weight of lithium required	2.5×10^{5} lbs.
Steam conditions	900°F, 850 psig
Overall plant cycle efficiency (less magnet)	34.8 %

III. Quantities pertaining to each section and overall for each value of maximum magnetic field.

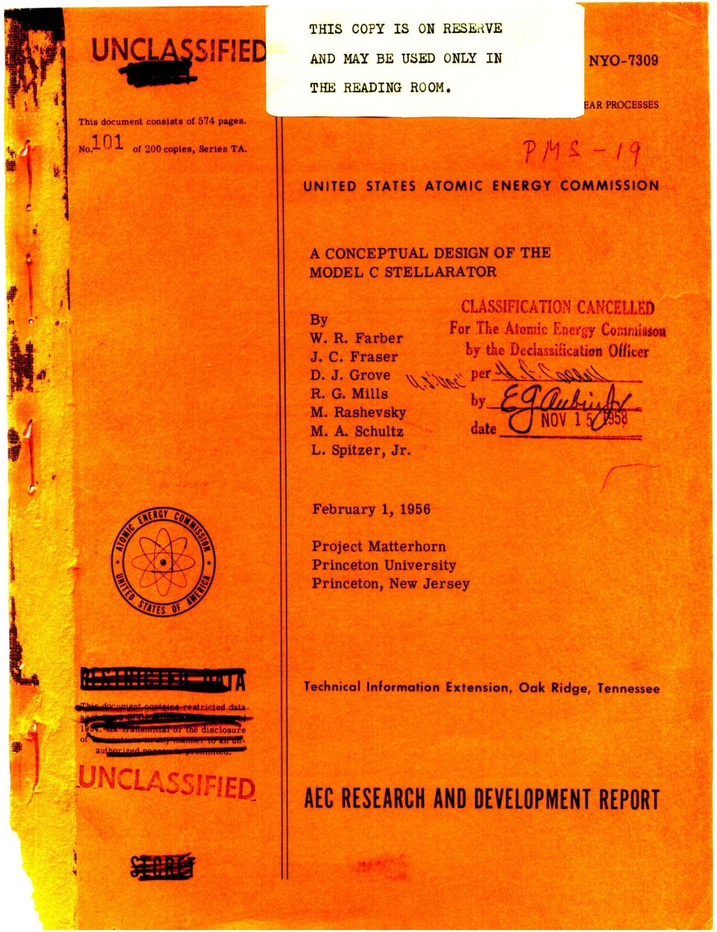
	Symbol	Units	Maximum Magnetic Field (Gauss) 50,000	75,000	100,000
Magnetic field at wall					
Positive curvature	B_{1w}	gauss	50,000	75,000	100,000
Negative curvature	B_{2w}	gauss	28,280	42,420	56,560
Straight sections	B_{3w}	gauss	28,280	42,420	56,560
Maximum current density in Cu (occurs at inside of curve of positive curvature section)		amp in^{-2}	10,700	16,100	21,400
Magnetic power required					
Positive curvature	P_{m1}	10^6 kw	0.315	0.697	1.24
Negative curvature	P_{m2}	10^6 kw	0.041	0.093	0.165
Straight sections	P_{m3}	10^6 kw	0.148	0.333	0.590
Divertor		10^6 kw	0.079	0.177	0.315
Total magnet power required	P_m	10^6 kw	0.578	1.30	2.31
Primary fast neutron flux					
Positive curvature	ϕ_1	$cm^{-2} sec^{-1}$	1.1×10^{13}	5.8×10^{13}	1.8×10^{14}
Negative curvature	ϕ_2	$cm^{-2} sec^{-1}$	1.7×10^{13}	8.5×10^{13}	2.7×10^{14}
Straight sections	ϕ_3	$cm^{-2} sec^{-1}$	1.7×10^{13}	8.5×10^{13}	2.7×10^{14}
Maximum heat absorbed in Li		cal $cm^3 sec^{-1}$	13.4	67.6	214
Maximum heat absorbed in H_2O		cal $cm^3 sec^{-1}$	26.6	135	426
Maximum ion density (center line)	n_{io}	cm^{-3}	8.65×10^{14}	1.95×10^{15}	3.46×10^{15}
Total number of ions	N_i	—	8.30×10^{22}	1.87×10^{23}	3.32×10^{23}
Minimum regenerative heating time	τ	sec	0.745	0.331	0.186
Tritium burn-up		kg/day	0.416	2.11	6.66
Tritium through-put		kg/day	24.11	122.2	386.8
Total nuclear power as heat	P_N	10^6 kw	3.41	17.3	54.5
Saleable electric power		10^6 kw	0.59	4.68	16.6

Model D – Tritium inventory considerations (1954)





Early Model C stellarator design (circa 1956)



UNCLASSIFIED
[redacted]

THIS COPY IS ON RESERVE
AND MAY BE USED ONLY IN
THE READING ROOM.

NYO-7309

EAR PROCESSES

This document consists of 574 pages.
No. 101 of 200 copies, Series TA.

PMS-19

UNITED STATES ATOMIC ENERGY COMMISSION

A CONCEPTUAL DESIGN OF THE
MODEL C STELLARATOR

By
W. R. Farber
J. C. Fraser
D. J. Grove
R. G. Mills
M. Rashevsky
M. A. Schultz
L. Spitzer, Jr.

CLASSIFICATION CANCELLED
For The Atomic Energy Commission
by the Declassification Officer
per
by
date NOV 15 1958

February 1, 1956

Project Matterhorn
Princeton University
Princeton, New Jersey

ATOMIC ENERGY COMMISSION
UNITED STATES OF AMERICA

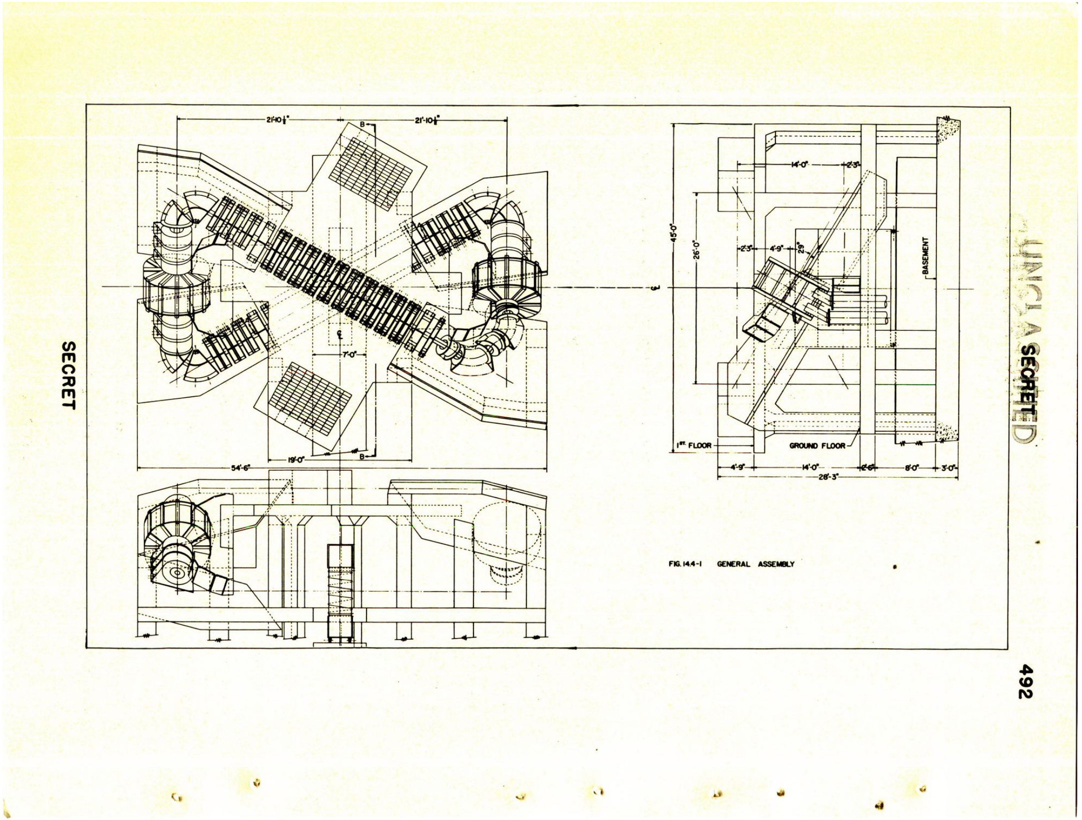
[redacted]

UNCLASSIFIED

Technical Information Extension, Oak Ridge, Tennessee

AEC RESEARCH AND DEVELOPMENT REPORT

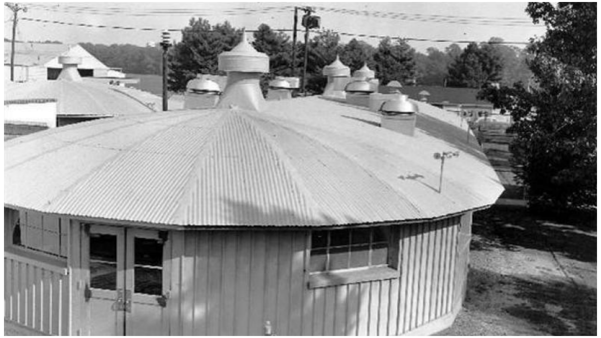
[redacted]



Fusion research declassified (1958)

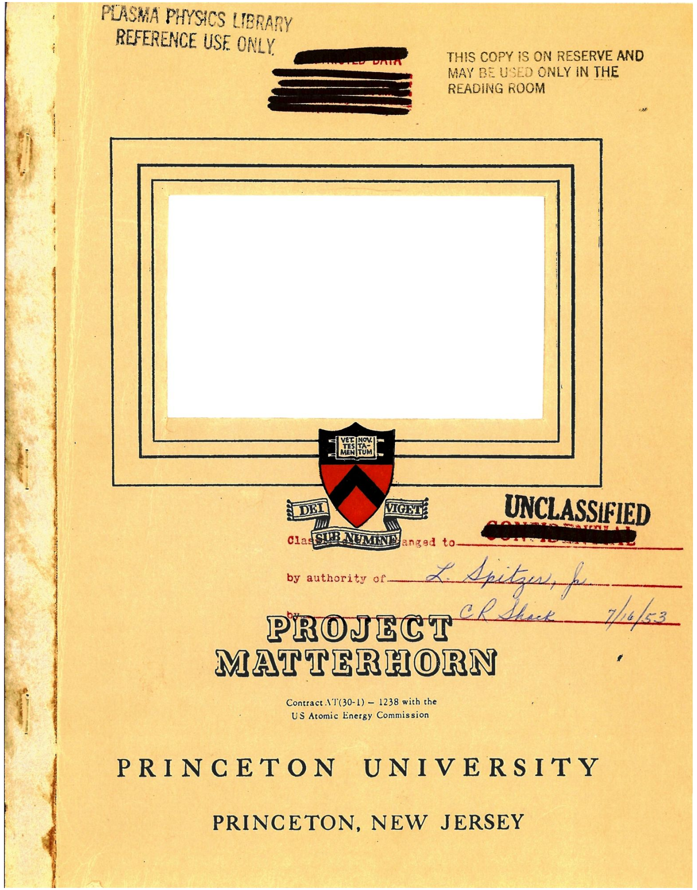
In 1958, Project Matterhorn was declassified:

- Spitzer was Director of Project Matterhorn (1951-1961) and founder of PPPL.
- In 1961, the Princeton Plasma Physics Laboratory was founded on the site.



Site of Project Matterhorn circa 1951

PPPL today



PLASMA PHYSICS LIBRARY
REFERENCE USE ONLY

[redacted]

THIS COPY IS ON RESERVE AND MAY BE USED ONLY IN THE READING ROOM

DEI SUB NUMINE VIGET

UNCLASSIFIED [redacted]

Clas... changed to

by authority of L. Spitzer, Jr.

by C R Shock 7/16/53

PROJECT MATTERHORN

Contract AT(30-1) – 1238 with the US Atomic Energy Commission

PRINCETON UNIVERSITY

PRINCETON, NEW JERSEY

Model C stellarator (1962-1969)

- The Model C (1962-1969) was PPPL's last operational stellarator:

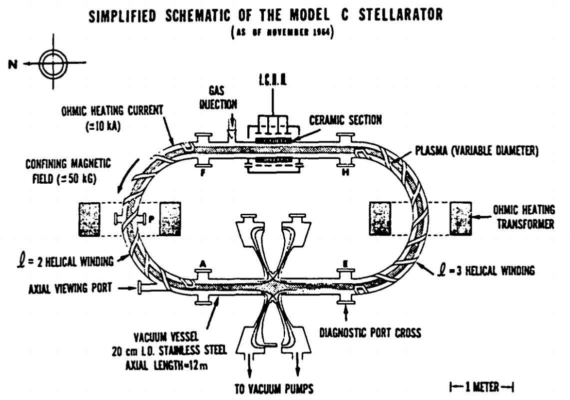


FIG.1. Top view of the Model C Stellarator. Two helical windings, a divertor, and an RF coil are in view. Two iron core transformers were used to induce a modest Ohmic heating current.

TABLE I. MODEL C STELLARATOR PARAMETERS

Total length (racetrack)	1200 cm
Minor radius	5–7.5 cm
B_t	10–50 (typically to 35) kG
Ohmic heating current	0–8 kA
RF heating power (at 25 MHz)	to 4 MW and to 5 ms
Density range	to 10^{14} cm^{-3}
Temperature	to 400 eV

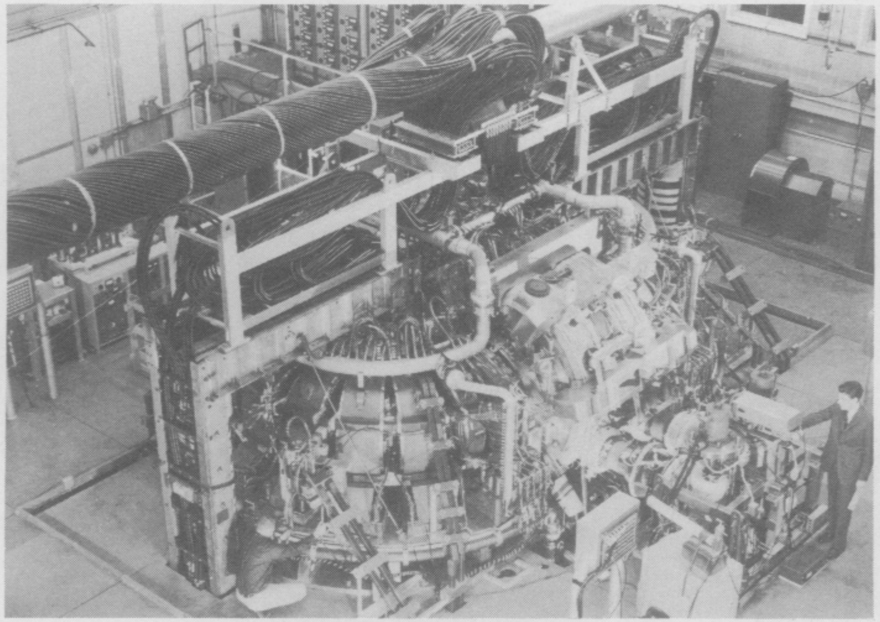


Fig. 3. The Model C Stellarator.

(From J. L. Johnson et al. IEEE Trans. Plasma Science PS-9.4 (1981))

Challenges for PPPL's early stellarators

- New measurement techniques developed and applied on the Model C stellarator confirmed the existence of magnetic surfaces and islands.

- However, the device was plagued by poor particle confinement.
- Concurrently, promising results redirected many research efforts in the US program towards tokamak configurations.
- At PPPL, the Model C stellarator was converted to the Symmetric Tokamak.

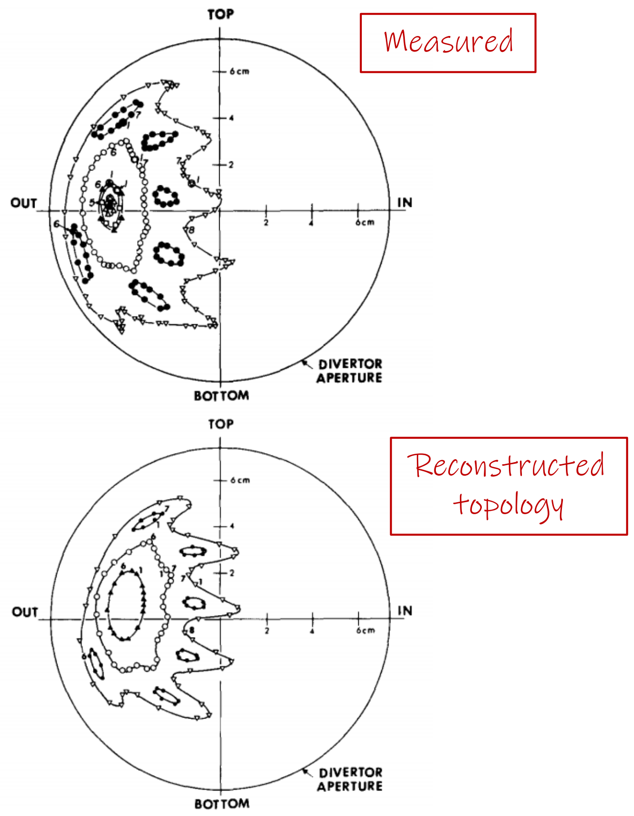


FIG. 2. Top curve: magnetic surfaces measured for various positions of the gun (encircled points marked "1"). Typical passes flanking the first are also numbered on each curve. A transverse magnetic field (1/400 of main field when averaged around the axis) was applied on one U bend. Bottom curve: computed magnetic surfaces for this condition.

(RHS: From R. M. Sinclair et al. Applied Physics Letters 17.2 (1970))

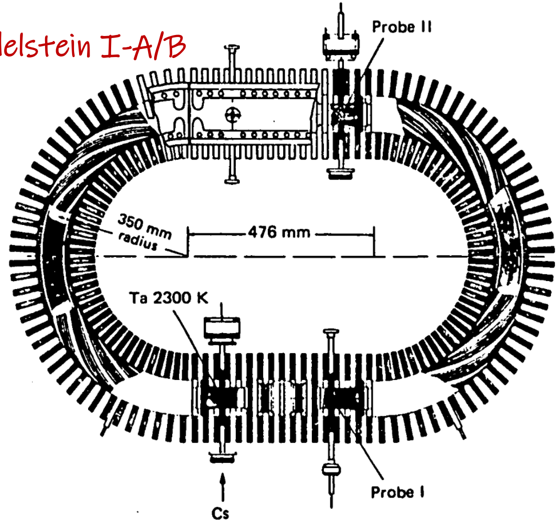
Early Wendelstein-series stellarators (circa 1960s)

Early research at the Max-Planck Institute in München (later Garching) produced the first members of the Wendelstein (W)-series of stellarators:

- W I-B reported good confinement of cesium plasmas.
- However, the results were highly sensitive to the configuration of correction fields.
- This was believed to be related to the racetrack configuration.

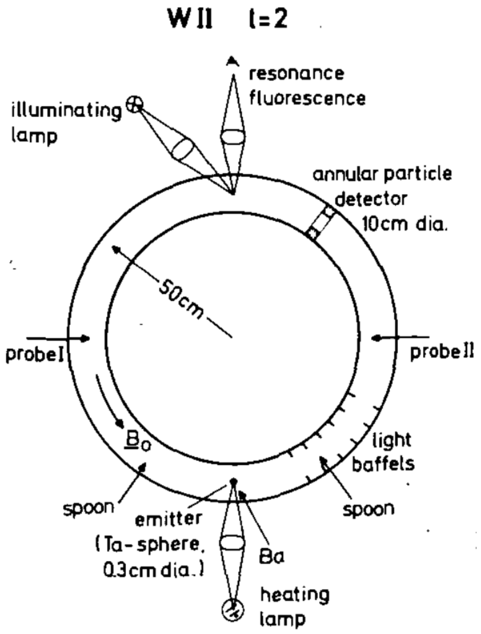
Wendelstein I-A (R_0=0.35 m, a~0.02 m, B<2 T)	Racetrack configuration ($\ell = 3$). Magnetic shear (~r^2) insufficient to provide equilibrium.
Wendelstein I-B (R_0=0.35 m, a~0.02 m, B<2 T)	Same dimensions as W I-A with $\ell = 2$ and higher ι_{axis}.

Wendelstein I-A/B



The extreme sensitivity of W I-B to correction fields motivated the pursuit of a circular torus for Wendelstein II-A:

Wendelstein II-A
R_0=0.5 m
A=0.06 m
$\ell = 2$, NFP=5
B~1.5 T (pulsed) from 44 equally spaced coils



(From E. Berkl et al., Proceedings of the Third International Conference on Plasma Physics and Controlled Nuclear Fusion Research. Vol. I. (1969).)

- Magnetic surfaces were measured using a pulsed electron beam.
- Experiments provided insight on the advantages of low magnetic shear and accurate construction of the coil system for avoiding magnetic islands.

Confinement and rotational transform

- W II-A had very low magnetic shear.
- Measurements of peak ion density in W II-A showed a strong dependence on whether ι was rational or irrational.

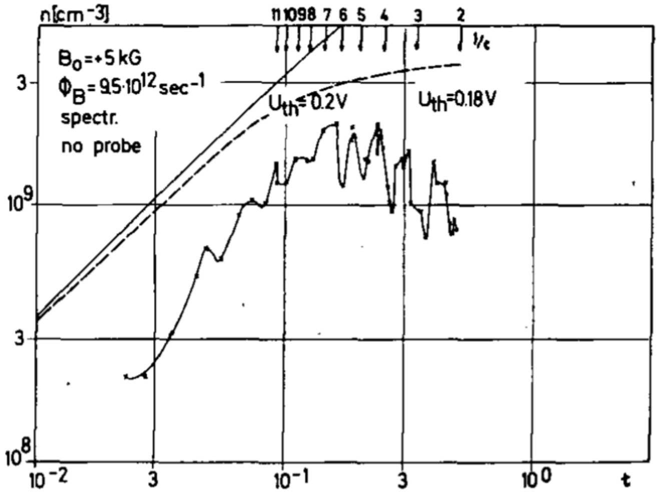


FIG. 8. Peak ion density vs. ι measured by probe for B_0 = 7.5 kG and $\Phi_B = 5.8 \times 10^{12}\ s^{-1}$. Calculated curves for U_{th} = 0.2 V and 0.18 V, respectively, are shown for comparison.

(From E. Berkl et al., Proceedings of the Third International Conference on Plasma Physics and Controlled Nuclear Fusion Research. Vol. I. (1969).)

Pfirsch-Schlüter currents

- The current density can be decomposed as:

$$\mathbf{J} = \mathbf{J}_\perp + J_\parallel \hat{\mathbf{b}}$$

- Force balance (i.e., ideal MHD equilibrium) requires:

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

- It follows that:

$$\mathbf{J}_\perp = \frac{\mathbf{B} \times \nabla p}{B^2}$$

- If $|J_\perp^A| > |J_\perp^B|$ for two points, A and B, on the same flux surface → charge separation.
- Maintaining quasi-neutrality ($\nabla \cdot \mathbf{J} = 0$) → Pfirsch-Schlüter current, $J_\parallel \neq 0$.

Parallel currents and ideal MHD equilibria

- The Pfirsch-Schlüter current produces a vertical magnetic field causing the plasma to shift outwards (Shafranov shift).
- Since the PS current depends on ∇p, the Shafranov shift limits maximum plasma β.

- Simultaneously enforcing $\mathbf{J} \times \mathbf{B} = \nabla p$ and $\nabla \cdot \mathbf{J} = \mathbf{0}$ yields:

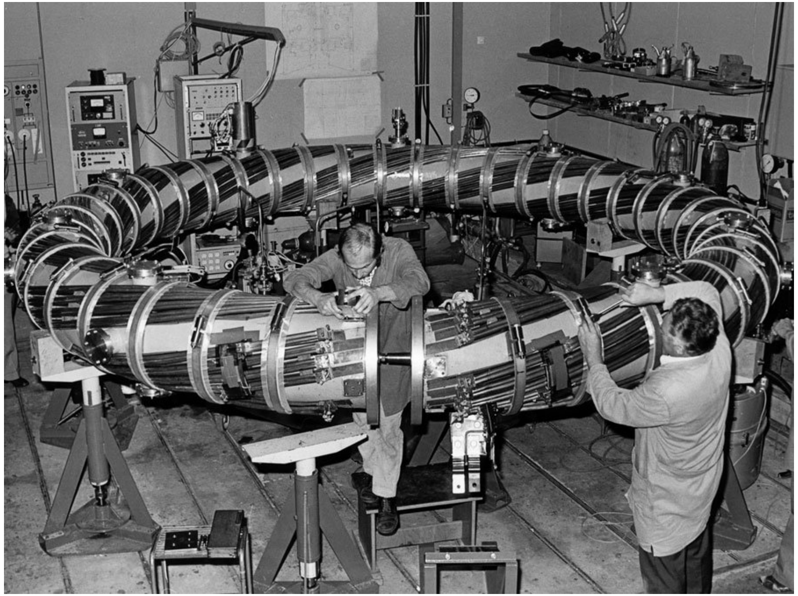
$$\mathbf{B} \cdot \nabla \left(\frac{J_{\parallel}}{B^2} \right) = -\nabla \cdot \left(\frac{\mathbf{B} \times \nabla p}{B^2} \right)$$

- Which is a magnetic differential equation for $J_{\parallel}$.
- In 3D, the solution for $J_{\parallel}$ contains contributions from PS and δ-function current densities and depends on ι. Can lead to island formation.
- This constrains the existence and uniqueness of ideal MHD equilibria in 3D [LMIG, Day 4 Thursday 11:30am].

Wendelstein 7-A (1975-1985)

Compared to WI/II stellarators, W7-A was a considerably larger device:

- Importantly, in 1980 W7-A successfully demonstrated confinement without plasma current.

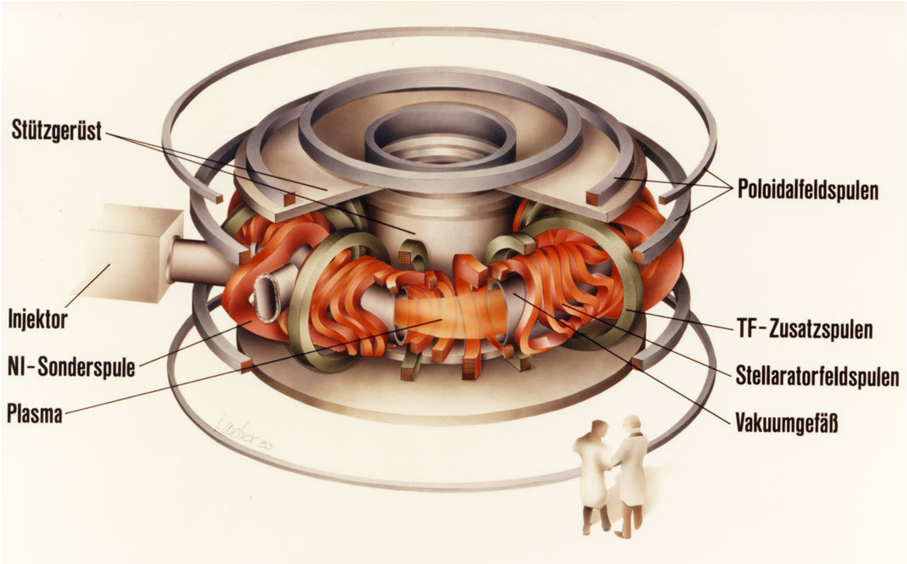


Major plasma radius	2 m
Minor plasma radius	0.12 m
Magnetic field	3.4 T
Number of coils	40 (+ helical winding)
Plasma volume	0.6 m^3
Pulse length	0.15 s
Plasma heating	2.9 MW

(From https://www.ipp.mpg.de/3951949/wendelstein7a)

Optimised stellarators: Wendelstein 7-AS (1988-2002)

W7-AS was an important advance for the stellarator program as it was the first to be designed using optimsation techniques:

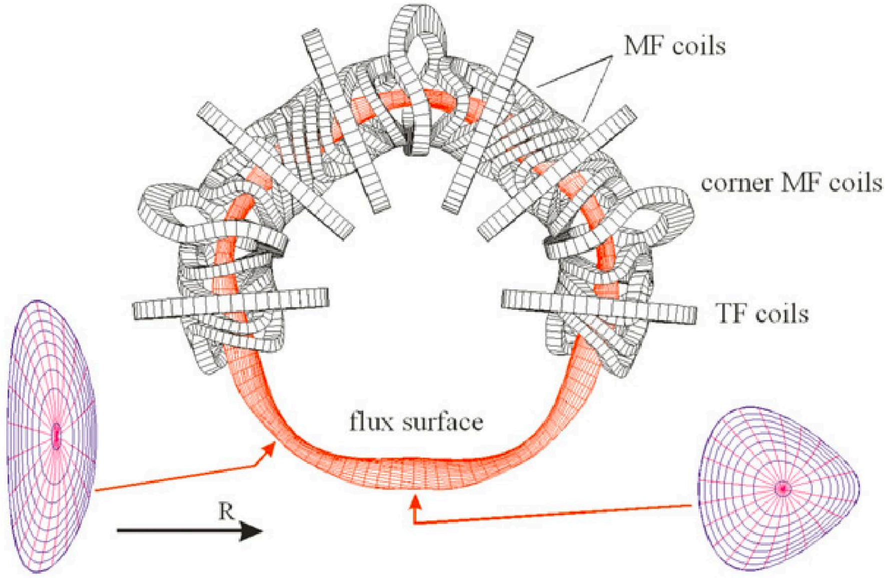


Major plasma radius	2 m
Minor plasma radius	0.2 m
Magnetic field	2.5 – 3.5 T
Number of coils	45
Plasma volume	1 m^3
Pulse length	3 s

(From https://www.ipp.mpg.de/3951975/w7as)

Design principles for Wendelstein 7-AS (1988-2002)

- Following W7-A, W7-AS was designed to minimise the equilibrium Pfirsch-Schlüter currents.
- Modular coils were designed to provide the requisite confining field with a fixed boundary optimisation approach.



W7-AS: A proof-of-concept for optimised stellarators

The physics objectives for W7-AS included:

- Demonstrating the modular coil concept.
- Investigation of a currentless plasma.
- Development of the island divertor concept.

W7-AS was a resounding success:

- 56953 discharges, ~14 years
- 50% reduction in parallel currents compared to W7-A (unoptimised).
- Improved guiding center confinement.
- Reduction of intercoil forces in modular coils.

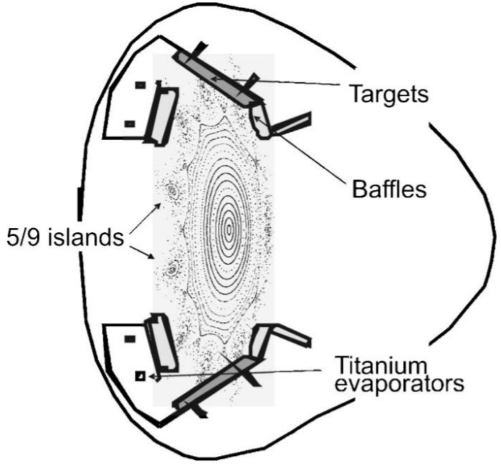


FIG. 3. Cross section in the elliptical plane of W7-AS: shown are the vacuum flux surfaces limited by the separatrix of the 5/9 island chain and two up–down divertor modules with targets, baffles, and titanium evaporators.

(From F. Wagner et al., Physics of Plasmas 12.7 072509 (2005)).

These insights directly informed the design of the world's current leading optimised stellarator, W7-X.

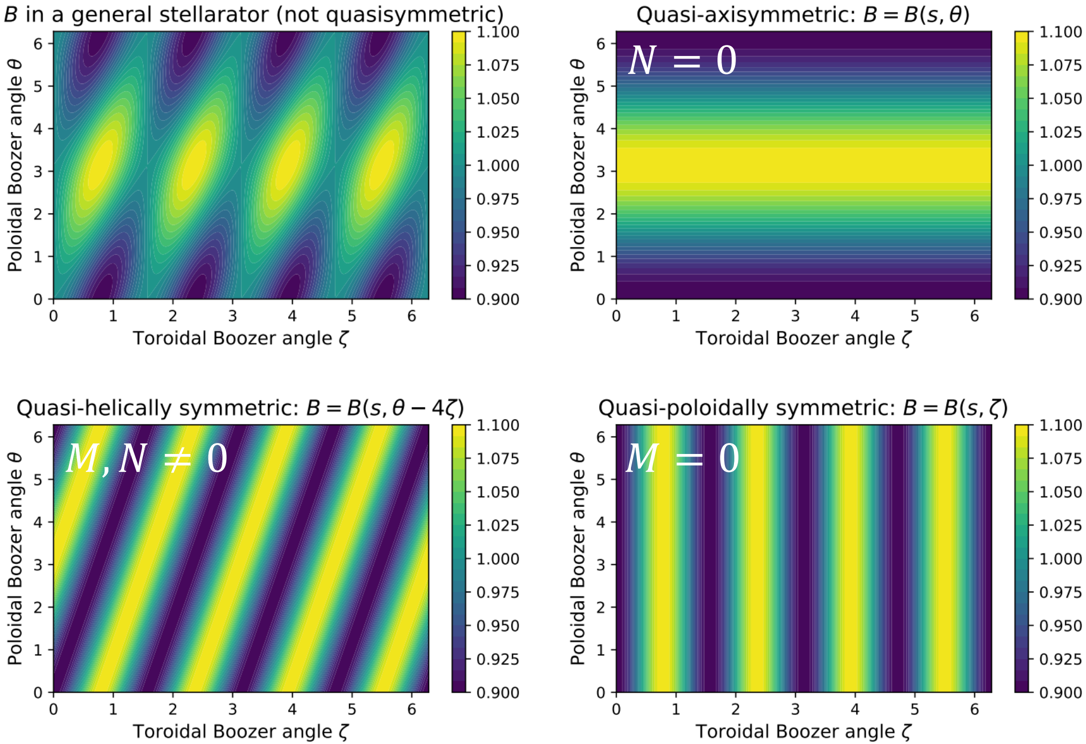
Symmetry and particle confinement

- Early stellarators suffered from poor particle confinement; a major obstacle for any fusion energy device.
- Symmetry breaking leads to the loss of a conserved quantity. When $\partial\phi \nrightarrow \mathbf{0}$, the guiding center orbits of trapped particles drift radially, leading to losses.
- Moreover, these losses are enhanced by collisions between charged particles, making neoclassical transport the dominant loss channel.

- Theoretical developments in 1980s identified a transformation to a coordinate system (Boozer coordinates) where guiding center drift orbits and neoclassical transport have a special property, i.e., depending only on $|\mathbf{B}|$ and not $\mathbf{B}$ on each flux surface.
- Confinement can be improved by designing $\mathbf{B}$ s.t. $|\mathbf{B}|$ has a special symmetry in this coordinate system → quasisymmetry [EJP, Day 5 1pm].

In Boozer coordinates $\{\psi, \vartheta_B, \zeta_B\}$ for fixed (M, N):

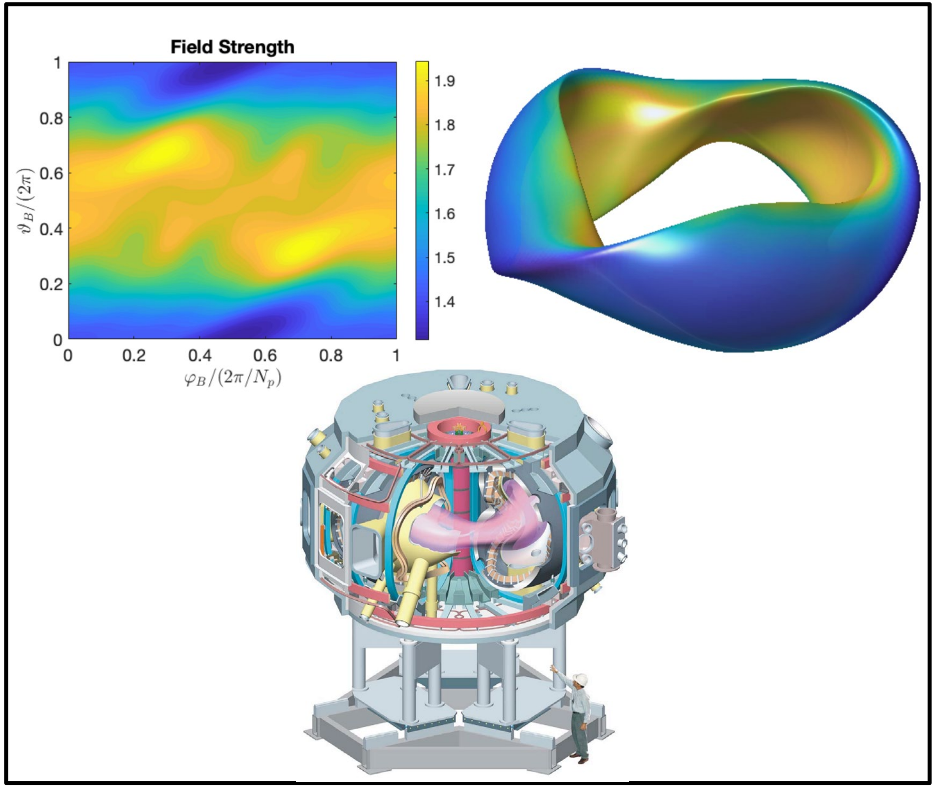
$$B(\psi, \vartheta_B, \zeta_B) = B(\psi, M\vartheta_B - N\zeta_B)$$



(From M. Landreman, Quasisymmetry: A hidden symmetry of magnetic fields (2019)).

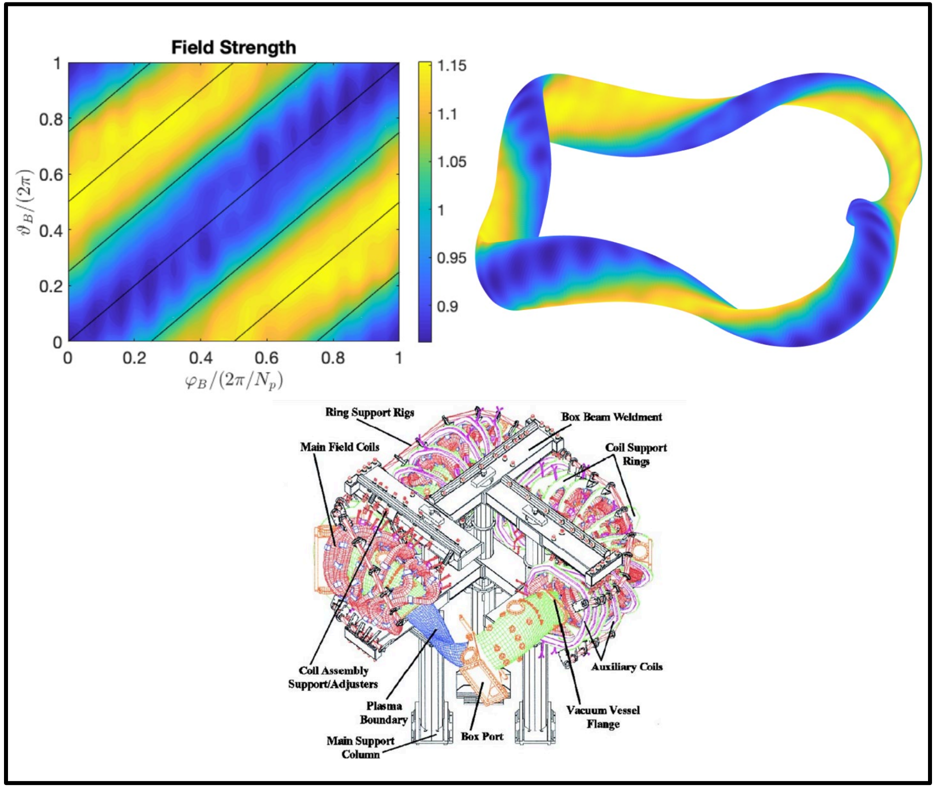
Example: Quasisymmetric devices

NCSX (quasi-axisymmetric):
$N = 0$



(From NCSX Annual Highlights (2006)).

HSX (quasihelical):
$M \neq 0, N \neq 0$



(From A. F. Almagri et al., IEEE Trans. Plasma Sci. 27.1 (1999)).

Omnigeneity: Particle confinement with collisions

- Omnigeneity is a property whereby the time-averaged guiding center drift off a magnetic surface vanishes for all particles and important for confinement in stellarators.

- Quasisymmetry confines particles in the absence of collisions, it is sufficient but not necessary for confinement in a stellarator, as it implies omnigeneity.
- However, omnigeneity includes a much wider class of magnetic fields than quasisymmetry.
- Notably, a magnetic field which is arbitrarily close to omnigeneity be very far from quasisymmetry.

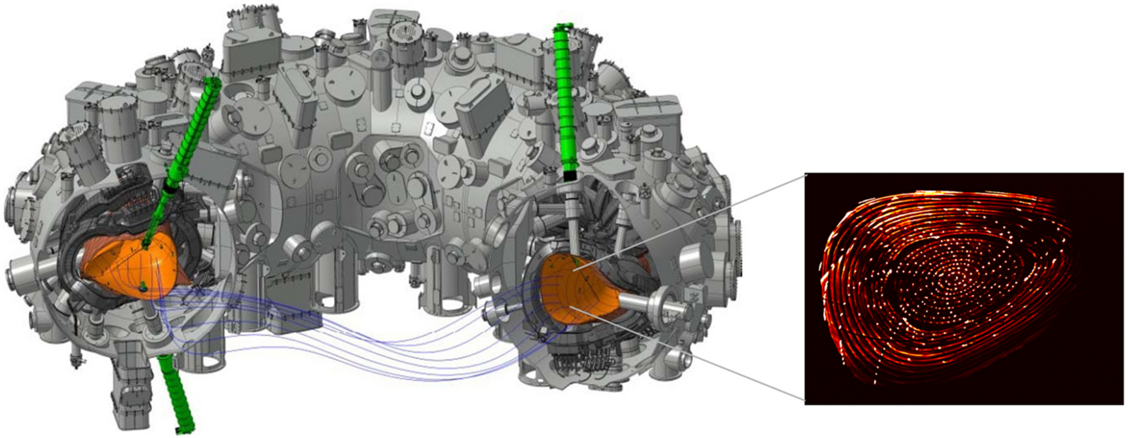
Design principles for Wendelstein 7-X (2015+)

The design of W7-X was developed by optimisation under several criteria including:

- Nested flux surfaces without major resonances and only small islands.
- Low Shafranov shift toward high β.
- Good MHD stability properties with a stability limit close to $\langle\beta\rangle \approx 5\%$.
- Low neoclassical losses in the low collisionality regime.
- Low bootstrap current in the low collisionality regime to maintain the field optimization from low to high β.

(From F. Wagner et al., Physics of Plasmas 12.7 072509 (2005)).

W7-X is a 5-field period, large-aspect ratio device that operates with low shear, minimal current and quasi-omnigenous magnetic field:



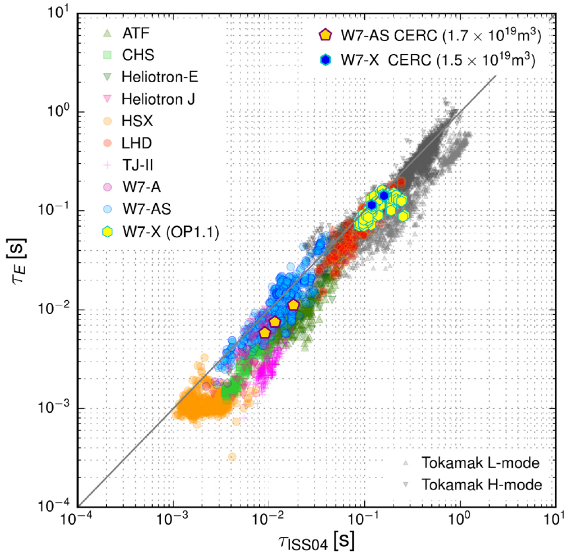
Major plasma radius	5.5 m
Minor plasma radius	0.5 m
Magnetic field	3 T
Number of coils	50 modular non-planar and the 20 planar superconducting NbTi coils
Plasma volume	30 m^3
Pulse length	3 s (up to 1800 s)

(From R. C. Wolf et al., Nuclear Fusion 57.10 (2017)).

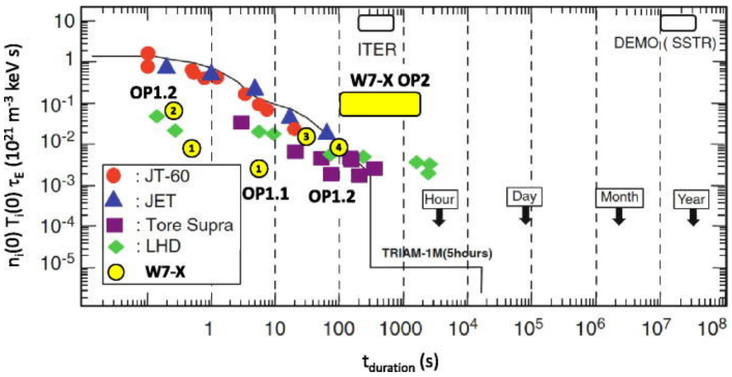
W7-X construction montage

W7-X began operations in December 2015 and has been a highly successful demonstration of modern stellarator optimization:

- With effective suppression of neoclassical transport, losses due to turbulent transport have become an important target for next-generation optimised designs.
- With the installation of actively-cooled divertors, high-β and long-pulse (~30 min) operation is planned for the upcoming experimental campaign.



(From R. C. Wolf et al., Nuclear Fusion 57.10 (2017)).



(From R. C. Wolf et al., Physics of Plasmas 26.8 (2019)).

Precision construction in W7-X

- The W7-X non-planar coils are geometrically complex, construction took >10 years and experienced challenges including:
  - Delays in component deliveries
  - Quality deviations in major components
  - A significant increase in design and assembly effort
  - Lack of engineering capacity
  - Inadequate managerial processes
  - Lack of project-oriented work style

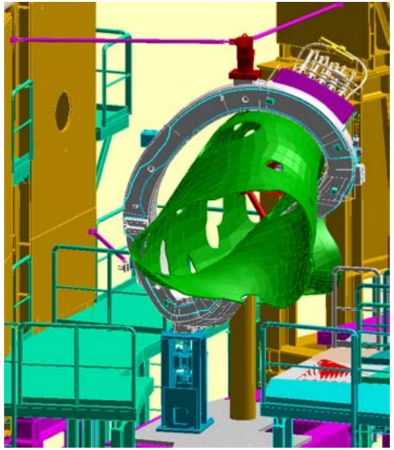


Fig. 6: Alignment of a single coil in first assembly step (CAD-drawing). Coil is fixed on a foot column in the right position. With both upper and one horizontal (left side) bar the coil is aligned to nominal position.

- Typical alignment tolerances ~1.5 mm.
- For example, for two halves of a magnet module: 0.05 mm for the maximum remaining gap between two surfaces and 0.5 mm for the maximum lateral shift.

(Figure from T. Braeuer et al., IEEE 25th Symposium on Fusion Engineering (SOFE) (2013)).

Design and project management lessons learned from W7-X

Lessons learned from the W7-X project inform the design principles for next-generation optimised stellarators:

Lesson 1: Limited margins, clearances and low tolerance levels

- Increases complexity and risk, impacting schedule, budget and capacity (e.g., through late-stage design changes).

Lesson 2: Robust QA framework in place for major components prior to tender

- The actual manufacturing must be accompanied by a dedicated test program.

Lesson 3: Development and manufacturing risks borne by project

- The low risk appetite of industry/manufacturing partners means associated costs are shifted onto project, potentially impacting budget.

- **Outcome:** Future designs need to give simultaneous consideration to engineering and physics optimisation.

(From T. Klinger, Fusion Engineering and Design 88.6-8 (2013)).

Techniques for stellarator design

Design principles

The design criteria for a fusion device are not limited to plasma properties. It must also be buildable within economic constraints:

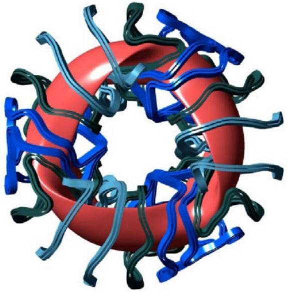
- This is difficult and can be limited by technology and science knowledge.

Physics → Engineering - - → Construction

MHD equilibrium

↓

Coils



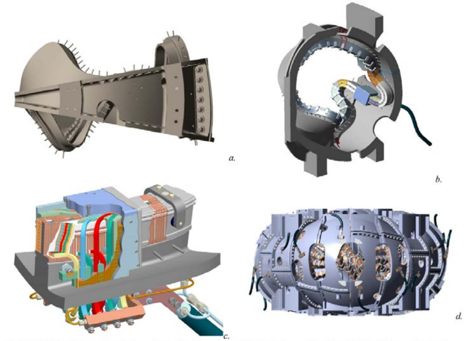


Fig. 2. NCSX modular coil set design. (a) winding form, (b) coil, (c) winding pack and lead details, (d) completed coil set..

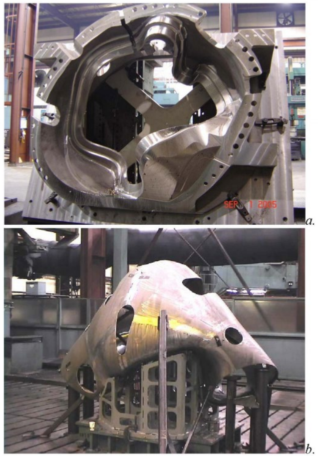


Fig. 4. NCSX components in manufacture: (a) MCWF, (b) vacuum vessel.

(From H. G. Neilson et al., 21st IEEE/NPS Symposium on Fusion Engineering (SOFE) (2005)).

Developing design criteria

Stellarator physics is difficult to model:

- High-fidelity physics simulations and whole-device modelling inform the development of physics targets for optimisation.
- But they are computationally expensive and time consuming so cannot be used directly.
- Since optimisation requires fast evaluation of figures-of-merit, reduced models are typically required.

Good physics properties do not imply buildability:

- Incorporating engineering constraints in the physics design is important.

Modern frameworks for stellarator design

Integrated systems optimisation

Stellarator optimisation

Physics ⇄ Systems (0/1D) → Engineering → Construction

MHD equilibrium ⇅ Coils

Tools include:
SIMSOPT
STELLOPT
ROSE
lasso.jl

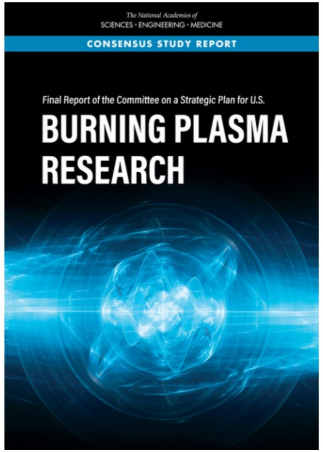
Tools include:
PROCESS

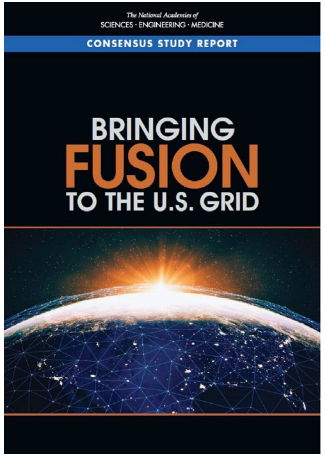
More on stellarator optimisation on Friday [EJP, Day 5 1pm]

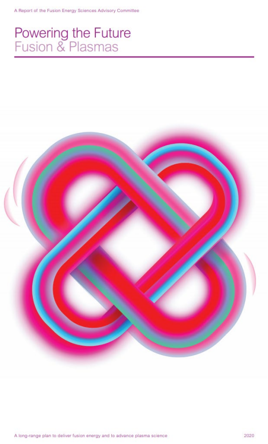
Towards a stellarator pilot plant

Towards a stellarator pilot plant

A recent series of reports have emphasised the need for progress towards a pilot fusion power plant…







…stellarators have an important role to play.

– The plan embraces the development of innovative ideas that could lead to more commercially attractive fusion systems and address critical gaps. The quasi-symmetric stellarator is the leading US approach to developing disruption-free, low-recirculating-power fusion configurations and should be tested experimentally with a new US stellarator facility.

PRINCETON PLASMA PHYSICS LABORATORY

Thank you!

: @mini_space_dino

Email: awright@pppl.gov